9.5a Alternating Series Test (AST) p. 625 5-26 all

Alternating Series: series whose terms alternate signs

Alternating Series Test: Let $a_n > 0$

The alternating series: $\sum_{n=1}^{\infty} (-1)^n a_n$ and $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ will converge if:

1) $\lim_{n\to\infty} a_n = 0$ and 2) $a_{n+1} \le a_n$ for all n.

Determine convergence/divergence of series

6) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3n+2}$ $\quad \lim_{n\to\infty} \frac{n}{3n+2} = \frac{1}{3} \neq 0$ Diverges by n^{th} term test.

$a_{n+1} \le a_n$

8) $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$ $\quad \lim_{n\to\infty} \frac{1}{e^n} = 0$, converges by AST (and GST since $\left(\frac{-1}{e}\right)^n$ is geometric series)

$a_{n+1} \le a_n$

10) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+5}$ $\quad \lim_{n\to\infty} \frac{n}{n^2+5} = 0$, $a_{n+1} \le a_n$ converges by AST

12) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ $\quad \lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$ $\quad a_{n+1} \le a_n$ Converges by AST

14) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{n^2+4}$ $\quad \lim_{n\to\infty} \frac{n^2}{n^2+4} = 1 \neq 0$, diverges by n^{th} term test.

16) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n+1)}{n+1}$ $\quad \lim_{n\to\infty} \frac{\ln(n+1)}{n+1} \xrightarrow{L'H} \frac{\frac{1}{n+1}}{1}$ $\quad \lim_{n\to\infty} \frac{1}{n+1} = 0$, $a_{n+1} \le a_n$

converges by AST.

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18) $\sum_{n=1}^{\infty} \frac{1}{n} \cos(n\pi) = \sum_{n=1}^{\infty} \frac{1}{n}(-1)^n$ $\quad \lim_{n\to\infty} \frac{1}{n} = 0$ $\quad$ Converges by AST.

$a_{n+1} \le a_n$

20) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$ $\lim_{n\to\infty} \frac{1}{(2n+1)!} = 0$ $a_{n+1} \le a_n$

Converges by AST

22) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$ $\lim_{n\to\infty} \frac{n^{1/2}}{n^{1/3}} \neq 0$, diverges by n^{th} term test.

24) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1\cdot 3\cdot 5 \cdots (2n-1)}{1\cdot 4\cdot 7 \cdots (3n-2)} = \lim_{n\to\infty} 3 \cdot \frac{5}{4} \cdot \frac{7}{7} \cdot \frac{9}{10} \cdot \frac{2n-1}{3n-5} \cdot \frac{1}{3n-2} = \boxed{0}$, $a_{n+1} \le a_n$

$\frac{3\cdot 5\cdot 7\cdot 9\cdot (2n-1)}{4\cdot 7\cdot 10\cdot (3n-5)} \cdot \frac{}{(3n-2)}$

Converges by AST

26) $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{e^n + e^{-n}} = \sum_{n=1}^{\infty} (-1)^{n+1} \operatorname{sech} n$ (hyperbolic trig)

$\frac{2}{e^n + \frac{1}{e^n}} = \frac{2}{\frac{e^{2n}+1}{e^n}} = \lim_{n\to\infty} \frac{2e^n}{e^{2n}+1} = \frac{\infty}{\infty} \xrightarrow{L'H} \frac{\cancel{2}e^n}{\cancel{2}e^{2n}} = \lim_{n\to\infty} \frac{1}{e^n} = 0$

$a_{n+1} < a_n$

Converges by AST