

## 9.56 Error Bounds / Absolute and Conditional Convergence

p. 625 27-54, 71-80

Approximating sum of series using first 6 terms (Error for Alt. Series that converge is the first unused term)

$$28) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 4}{\ln(n+1)} \quad \sum_{n=1}^6 \frac{4(-1)^{n+1}}{\ln(n+1)} \approx 2.7067$$

$$R_6 = |S - S_6| \leq a_7 \quad a_7 = \frac{4}{\ln 8} \approx 1.9236$$

$$2.7067 - 1.9236 \leq S \leq 2.7067 + 1.9236$$

$$\boxed{0.7831 \leq S \leq 4.6303}$$

$$30) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot n}{3^n} \rightarrow \sum_{n=1}^6 \frac{(-1)^{n+1} n}{3^n} \approx 0.1852$$

$$|R_6| = |S - S_6| \leq a_7 \quad a_7 = \frac{7}{3^7} \approx 0.0032$$

$$0.1852 - 0.0032 \leq S \leq 0.1852 + 0.0032$$

$$\boxed{0.1820 \leq S \leq 0.1884}$$

Determine # of terms needed to approximate sum of series w/ error less than 0.001

$$32) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \quad |R_n| \leq a_{n+1} \quad \frac{1}{(n+1)^2} \leq 0.001 \quad (N+1)^2 = \frac{1}{0.001} = 1000$$

$$(N+1)^2 = 1000 \quad N = 31$$

$$34) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5} \quad |R_n| \leq a_{n+1} \quad \frac{1}{(n+1)^5} \leq 0.001$$

31 terms needed

$$(n+1)^5 \leq \frac{1}{0.001} = 1000$$

$$(n+1)^5 \leq 1000$$

n = 3 terms

$$36) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \quad |R_N| \leq a_{n+1} \quad \frac{1}{(2(n+1))!} = \frac{1}{(2n+2)!} \leq 0.001$$

$$(2n+2)! = \frac{1}{0.001} = 1000$$

$N=3$ , use 4 terms  
(begins with  $n=0$ )

Determine Absolute/Conditional Convergence

$$38) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \quad \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ is a convergent } p\text{-series}$$

Absolute convergence

1) Absolute convergence  
if  $\sum |a_n|$  converges

2) Conditional convergence  
if  $\sum a_n$  converges but  
 $\sum |a_n|$  diverges

$$40) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$$

$$\text{let } a_n = \frac{1}{n+3}$$

$$b_n = \frac{1}{n} \left( \begin{array}{l} \text{diverges} \\ \text{by } p\text{-series} \end{array} \right)$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{n+3}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{n+3} \cdot n = \left[ \frac{1}{\infty} \right] = 0$$

diverges by Limit Comparison Test

Conditional convergence

$$42) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}} = \frac{(-1)^{n+1}}{n^{3/2}}$$

since  $\frac{1}{n^{3/2}}$  converges by  $p$ -series ( $p = 3/2 > 1$ )

Absolute convergence

$$44) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+3)}{n+10}$$

$$\lim_{n \rightarrow \infty} \frac{2n+3}{n+10} = 2 \neq 0 \quad \text{diverges by } n^{\text{th}} \text{ term test.}$$

$$46) \sum_{n=0}^{\infty} \frac{(-1)^n}{e^{n^2}}$$

$$\left(\frac{1}{e}\right)^{n^2} < \left(\frac{1}{e}\right)^n \rightarrow \text{converges by GST}$$

By DCT, series converges absolutely

$$48) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{4/3}}$$

$\frac{1}{n^{4/3}}$  converge by p-series test ( $p = 4/3 > 1$ )

Series converges absolutely

$$50) \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$$

let  $a_n = \frac{1}{\sqrt{n+4}}$

let  $b_n = \frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$  (divergent p-series)  
 $p = 1/2 < 1$

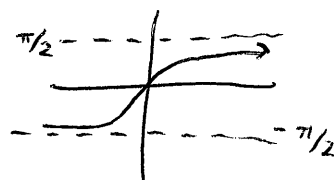
$$\lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n+4}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+4}} \cdot \frac{\sqrt{n}}{1} = 1$$

Diverges by Limit Comparison Test

Conditional convergence

$$52) \sum_{n=1}^{\infty} (-1)^{n+1} \arctan n \quad \lim_{n \rightarrow \infty} \arctan n = \pi/2 \neq 0$$

Diverges by  $n^{\text{th}}$  term test



$$54) \sum_{n=1}^{\infty} \frac{\sin\left[\frac{(2n-1)\pi}{2}\right]}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

conditional convergence  
since  $\sum \frac{1}{n}$  diverges (harmonic p-series)

Determine convergence/divergence. Identify convergence test used:

$$71) \sum_{n=1}^{\infty} \frac{10}{n^{3/2}} \quad \text{convergent p-series } p = 3/2 > 1$$

$$72) \sum_{n=1}^{\infty} \frac{3}{n^2+5}$$

let  $a_n = \frac{3}{n^2+5}$

$b_n = \frac{1}{n^2}$  (convergent p-series)

$$\lim_{n \rightarrow \infty} \frac{\frac{3}{n^2+5}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{3}{n^2+5} \cdot \frac{n^2}{1} = \frac{3}{5}$$

Converges by Limit Comparison Test

$$73) \sum_{n=1}^{\infty} \frac{3^n}{n^2} \quad \lim_{n \rightarrow \infty} \frac{3^n}{n^2} = \infty \neq 0 \quad \text{Diverges by } n^{\text{th}} \text{ term test.}$$

$$74) \sum_{n=1}^{\infty} \frac{1}{2^{n+1}} \quad \frac{1}{2^{n+1}} < \frac{1}{2^n} \left( \begin{array}{l} \text{convergent} \\ \text{Geometric} \\ \text{series } r = \frac{1}{2} \end{array} \right) \quad \left| \begin{array}{l} \text{series converge by} \\ \text{Direct Comparison Test} \end{array} \right.$$

$$76) \sum_{n=1}^{\infty} \frac{3n^2}{2n^2+1} \quad \lim_{n \rightarrow \infty} \frac{3n^2}{2n^2+1} = \frac{3}{2} \neq 0 \quad \text{Diverges by } n^{\text{th}} \text{ term test}$$

$$75) \sum_{n=0}^{\infty} 5\left(\frac{7}{8}\right)^n \quad \text{By Geometric series test } (r = \frac{7}{8} < 1), \text{ series converge.}$$

$$77) \sum_{n=1}^{\infty} 100e^{-n/2} \quad 100\left(\frac{1}{e}\right)^{n/2} \quad 100\left(\frac{1}{\sqrt{e}}\right)^n \quad \text{Converges by geometric series test } r = \frac{1}{\sqrt{e}} < 1$$

$$78) \sum_{n=0}^{\infty} \frac{(-1)^n}{n+4} \quad a_n = \frac{1}{n+4} \quad \boxed{\text{AST}} \quad \left| \begin{array}{l} \lim_{n \rightarrow \infty} \frac{\frac{1}{n+4}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{n+4} \cdot \frac{n}{1} = 1 \\ \text{Series diverge by Limit Comparison Test} \\ \text{(conditional convergence)} \end{array} \right.$$

$$79) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 4}{3n^2-1} \quad \lim_{n \rightarrow \infty} \frac{4}{3n^2-1} = 0 \quad a_{n+1} \leq a_n \quad \text{Converges by Alt. Series Test (Absolute convergence)}$$

$$80) \sum_{n=2}^{\infty} \frac{\ln n}{n} \quad \frac{\ln n}{n} > \frac{1}{n} \left( \begin{array}{l} \text{harmonic} \\ \text{series} \end{array} \right) \quad \text{Diverges by Direct Comparison Test}$$

$$\int_2^{\infty} \frac{\ln x}{x} dx \quad \left| \begin{array}{l} u = \ln x \\ \frac{du}{dx} = \frac{1}{x} \\ dx = x du \end{array} \right. \quad \int \frac{u}{x} \cdot x du \quad \frac{u^2}{2} = \frac{1}{2}(\ln x)^2 \Big|_2^{\infty} = \infty \quad \text{Diverges by Integral Test}$$