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9.8 Power Series p.654 1-44

Find Radius of Convergence / Interval of Convergence (IOC)

a) Apply Ratio Test $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$

$-r < x-c < +r$

$c-r < x < c+r$

b) Identify form $|x-\underset{\text{center}}{c}| < \underset{\text{radius of convergence}}{r}$

c) put in form $c-r < x < c+r$ to find possible IOC $[(c-r, c+r)]$

d) Determine convergence/divergence at endpoints.

Find Radius of Convergence

6) $\sum_{n=0}^{\infty}(3x)^n$ $L = \lim_{n\to\infty}\left|\frac{(3x)^{n+1}}{(3x)^n}\right| = |3x| < 1 \rightarrow |x| < 1/3 \rightarrow |x-\overset{\text{center}}{0}| < \overset{\text{radius}}{1/3}$

$$\boxed{R = 1/3}$$

8) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{5^n}$ $L = \lim_{n\to\infty}\left|\frac{\frac{x^{n+1}}{5^{n+1}}}{\frac{x^n}{5^n}}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{5^{n+1}}\cdot\frac{5^n}{x^n}\right| = \left|\frac{x}{5}\right| < 1$ $|x-\overset{\text{center}}{0}| < \overset{\text{radius}}{1/5}$

$$\boxed{|R| = 1/5}$$

10) $\sum_{n=0}^{\infty}\frac{(2n)!x^{2n}}{n!}$

$$L = \lim_{n\to\infty}\left|\frac{(2(n+1))!\,x^{2(n+1)}}{(n+1)!}\cdot\frac{n!}{(2n)!\,x^{2n}}\right|$$

$$L = \lim_{n\to\infty}\left|\frac{(2n+2)!\,x^{2n+2}}{(n+1)!}\cdot\frac{n!}{(2n)!\,x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{(2n+2)(2n+1)\cancel{(2n)!}\,x^2\,\cancel{n!}}{(n+1)\cancel{(n!)}\cancel{(2n)!}}\right| = \lim_{n\to\infty}\frac{4n^2}{n}x^2 = \infty$$

series only converges at center $x=0$

$R=0$

Find Interval of Convergence

12) $\sum_{n=0}^{\infty} (2x)^n$ $\quad L = \lim_{n\to\infty} \left| \frac{(2x)^{n+1}}{(2x)^n} \right| = |2x| < 1 \quad |x - \overset{\substack{\text{center}\\ \downarrow}}{0}| < \overset{\substack{\text{radius}\\ \swarrow}}{1/2}$

$$-1/2 < x < 1/2$$

*endpoints will diverge since this is geometric series

$$\boxed{\text{I.O.C} : (-1/2, 1/2) \text{ or } -1/2 < x < 1/2}$$

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14) $\sum_{n=0}^{\infty} (-1)^{n+1} (n+1) x^n$ $\quad L = \lim_{n\to\infty} \left| \frac{(n+1+1)x^{n+1}}{(n+1)x^n} \right| = |x| < 1 \quad |x - \overset{\substack{\text{center}\\ \downarrow}}{0}| < \overset{\substack{\text{radius}\\ \swarrow}}{1}$

$$-1 < x < 1$$

Possible IOC: $[(-1, 1)]$

Test endpts:

$x = -1$

$\sum_{n=0}^{\infty} (-1)^{n+1} (n+1)(-1)^n = -(n+1)$ diverges by n^{th} term test

Test $x = 1$

$\sum_{n=0}^{\infty} (-1)^{n+1} (n+1)(1)^n$

diverges by n^{th} term test

$$\boxed{\text{I.O.C} : (-1, 1)}$$

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16) $\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$ $\quad L = \lim_{n\to\infty} \left| \frac{(3x)^{n+1}}{(2(n+1))!} \cdot \frac{(2n)!}{(3x)^n} \right| = \lim_{n\to\infty} \left| \frac{(3x) \cdot \cancel{(2n)}!}{(2n+2)(2n+1)\cancel{(2n)}!} \right| = 0 < 1$

$$\boxed{\text{I.O.C. is } (-\infty, \infty)}$$

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18) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(n+1)(n+2)}$ $\quad \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+2)(n+3)}\cdot\frac{(n+1)(n+2)}{x^n}\right| = |x|<1 \quad . \quad |x-0|<1$

$-1<x<1$

Possible I.O.C $[(-1,1)]$

Test endpts:

$\underline{x=-1}$

$\sum_{n=0}^{\infty}\frac{(-1)^n(-1)^n}{(n+1)(n+2)} = \frac{1}{(n+1)(n+2)} < \frac{1}{n^2}$. converges by comparison test (convergent p-series)

$\underline{x=1}$

$\sum_{n=0}^{\infty}\frac{(-1)^n(1)^n}{(n+1)(n+2)}$ converges by Alt Series test

$\boxed{\text{I.O.C}: [-1,1] \text{ or } -1\le x\le 1}$

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20) $\sum_{n=0}\frac{(-1)^n n!(x-5)^n}{3^n}$ $\quad L=\lim_{n\to\infty}\left|\frac{(n+1)!(x-5)^{n+1}}{3^{n+1}}\cdot\frac{3^n}{n!(x-5)^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(x-5)}{3}\right| = \infty$

$\boxed{\text{I.O.C = none, series converges only at } x=5}$

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22) $\sum_{n=0}^{\infty}\frac{(x-3)^{n+1}}{(n+1)4^{n+1}}$ $\quad L=\lim_{n\to\infty}\left|\frac{(x-3)^{n+2}}{(n+2)4^{n+2}}\cdot\frac{(n+1)4^{n+1}}{(x-3)^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{(x-3)(n+1)}{(n+2)\cdot 4}\right| = \left|\frac{x-3}{4}\right|<1$

$|x-3|<4$

$-4<x-3<4$

$-1<x<7$

potential I.O.C $[(-1,7)]$

$|x-3|<4$

center

radius

$\underline{\text{Test } x=7}$

$\sum_{n=0}^{\infty}\frac{(4)^{n+1}}{(n+1)4^{n+1}} = \frac{1}{n+1}$ diverges by Integral Test, Limit Comparison Test

$\underline{\text{Test } x=-1}$

$\sum_{n=0}^{\infty}\frac{(-4)^{n+1}}{(n+1)4^{n+1}} = \frac{(-1)^{n+1}}{n+1}$ converges by Alt. Series Test

$\boxed{\text{I.O.C}: [-1,7) \quad -1\le x<7}$

24) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n2^n}$ $\quad L = \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)2^{n+1}}\cdot\frac{n\cdot 2^n}{(x-2)^n}\right| = \lim_{n\to\infty}\left|\frac{(x-2)}{2}\cdot\frac{n}{n+1}\right| = \left|\frac{x-2}{2}\right| < 1$

$\left|\frac{x-2}{2}\right| < 1$

$|x-2| < 2$

center radius

$-2 < x-2 < 2$

$0 < x < 4$

potential I.O.C: $[(0,4)]$

Test endpts:

test $x=0$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-2)^n}{n\cdot 2^n} = \frac{(-1)^{2n+1}}{n} = \frac{-1}{n}$ divergent p-series

Test $x=4$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(2)^n}{n\cdot 2^n} = \frac{(-1)^{n+1}}{n}$ converges by Alt. series test

$\boxed{\text{I.O.C: } (0,4] \quad 0 < x \le 4}$

26) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$ $\quad L = \lim_{n\to\infty}\left|\frac{x^{2(n+1)+1}}{2(n+1)+1}\cdot\frac{2n+1}{x^{2n+1}}\right|$

$= \lim_{n\to\infty}\left|\frac{x^{2n+3}}{2n+3}\cdot\frac{2n+1}{x^{2n+1}}\right| = \lim_{n\to\infty}\left|x^2\cdot\frac{2n+1}{2n+3}\right| = |x^2| < 1 \quad -1 < x < 1$ possible IOC $[(-1,1)]$

Test $x=-1$

$\sum_{n=0}^{\infty}\frac{(-1)^n(-1)^{2n+1}}{2n+1} = \frac{(-1)^{3n+1}}{2n+1} = \frac{(-1)^{3n+1}}{2n+1}$ converges by Alt. series test

Test $x=1$

$\sum_{n=0}^{\infty}\frac{(-1)^n(1)^{2n+1}}{2n+1} = \frac{(-1)^n}{2n+1}$ converges by Alt. series test

$\boxed{\text{I.O.C. } [-1,1] \quad -1 \le x \le 1}$

28) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{n!}$ $\quad L = \lim_{n\to\infty}\left|\frac{x^{2(n+1)}}{(n+1)!}\cdot\frac{n!}{x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{x^{2n+2}\cdot\cancel{n!}}{(n+1)\cdot\cancel{n!}\,x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{x^2}{n+1}\right| = 0 < 1$

$\boxed{\text{I.O.C. is } (-\infty,\infty)}$

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30) $\displaystyle\sum_{n=1}^{\infty} \frac{n!\,x^n}{(2n)!}$ $\quad L = \lim_{n\to\infty}\left|\frac{(n+1)!\,x^{n+1}}{(2(n+1))!}\cdot\frac{(2n)!}{n!\,x^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)\cdot\cancel{n!}\cdot x\,\cancel{(2n)!}}{\cancel{n!}\cdot(2n+2)(2n+1)\cancel{(2n)!}}\right| = 0<1$

$\boxed{\text{I.O.C. is } (-\infty,\infty)}$

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32) $\displaystyle\sum_{n=1}^{\infty}\left[\frac{2\cdot4\cdot6\cdots2n}{3\cdot5\cdot7\cdots(2n+1)}\right]x^{2n+1}$

$\displaystyle\lim_{n\to\infty}\left|\frac{2\cdot\cancel{4\cdot6}\cdot\cancel{(2n)}(2n+2)\cdot x^{2n+3}}{\cancel{3\cdot5\cdot7}\cdots\cancel{(2n+1)}(2n+3)}\cdot\frac{3\cdot\cancel{5\cdot7}\cdots(2n+1)}{2\cdot\cancel{4\cdot6}\cdots\cancel{(2n)}\cdot x^{2n+1}}\right| = \lim_{n\to\infty}\left|\frac{2n+2}{2n+3}\cdot x^2\right| = |x^2|<1$

$-1 < x < 1$

possible I.O.C $[(-1,1)]$

Test $x=-1$

$\displaystyle\sum\left[\frac{2n}{2n+1}\right](-1)^{2n+1}$ diverges by n^{th} term test

Test $x=1$

$\displaystyle\sum\frac{2n}{2n+1}(1)^{2n+1}$ diverges by n^{th} term test

$\boxed{\text{I.O.C. } (-1,1) \quad -1<x<1}$

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34) $\displaystyle\sum_{n=1}^{\infty}\frac{n!\,(x+1)^n}{1\cdot3\cdot5\cdots(2n-1)}$ $\quad\displaystyle\lim_{n\to\infty}\left|\frac{(n+1)!\,(x+1)^{n+1}}{1\cdot\cancel{3\cdot5}\cdots\cancel{(2n-1)}(2n+1)}\cdot\frac{1\cdot\cancel{3\cdot5}\cdots\cancel{(2n-1)}}{n!\,(x+1)^n}\right|$

$\displaystyle = \lim_{n\to\infty}\left|\frac{(n+1)\cancel{n!}\,(x+1)}{\cancel{n!}\,(2n+1)}\right| = \left|\frac{1}{2}(x+1)\right|<1 \rightarrow |\underset{\text{center}}{x+1}| < \underset{\text{radius}}{2}$

$-2 < x+1 < 2$

$-3 < x < 1$

possible I.O.C: $[(-3,1)]$

Test $x=-3$

$\displaystyle\sum_{n=1}^{\infty}\frac{n!(-2)^n}{1\cdot3\cdot5\cdot(2n-1)}$ diverges by n^{th} term test

Test $x=1$

$\displaystyle\sum_{n=1}^{\infty}\frac{n!(2)^n}{1\cdot3\cdot5\cdots(2n-1)}$ diverges by n^{th} term test

$\boxed{\text{I.O.C. } (-3,1) \quad -3<x<1}$

*36) Find Radius of convergence

$$\sum_{n=0}^{\infty} \frac{(n!)^k x^n}{(kn)!}, \text{ k is positive integer} \quad \left| \lim_{n \to \infty} \left| \frac{[(n+1)!]^k x^{n+1}}{[k(n+1)]!} \cdot \frac{(kn)!}{(n!)^k \cdot x^n} \right| \right.$$

$$\lim_{n \to \infty} \left| \frac{[(n+1)]^k \cancel{[n!]^k} \cdot \cancel{x^n} \cdot x \cdot (kn)!}{(kn+k)(kn)(kn-k)(kn-2k) \cdot \cancel{(n!)^k} \cdot \cancel{x^n}} \right|$$

$$= \frac{|x|}{k^k} < 1 \qquad |x| < k^k \qquad \boxed{R = k^k}$$

38) Find Interval of convergence

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-c)^n}{nc^n} \quad \lim_{n \to \infty} \left| \frac{(x-c)^{n+1}}{(n+1) \cdot c^{n+1}} \cdot \frac{n \cdot c^n}{(x-c)^n} \right| = \lim_{n \to \infty} \left| \frac{(x-c)}{c} \cdot \frac{n}{n+1} \right| = \left| \frac{x-c}{c} \right| < 1$$

$$|x-c| < c$$

$$-c < x-c < c$$

$$0 < x < 2c$$

possible I.O.C. $[(0, 2c)]$

Test $x=0$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(0-c)^n}{nc^n} = \frac{(-1)^{2n+1}}{n} = \frac{-1}{n}$$ diverges by n^{th} term test

Test $x=2c$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2c-c)^n}{n \cdot c^n} = \frac{(-1)^{n+1}}{n}$$ converges by Alt. series test

$$\boxed{\text{I.O.C. } (0, 2c] \quad 0 < x \leq 2c}$$

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40) $\sum_{n=1}^{\infty} \frac{n!(x-c)^n}{1\cdot 3\cdot 5\cdots(2n-1)}$ $\lim_{n\to\infty}\left|\frac{(n+1)!(x-c)^{n+1}}{1\cdot\cancel{3}\cdot\cancel{5}\cdots\cancel{(2n-1)}(2n+1)}\cdot\frac{1\cdot\cancel{3}\cdot\cancel{5}\cdot\cancel{(2n-1)}}{n!(x-c)^n}\right|$

$\lim_{n\to\infty}\left|\frac{(n+1)(\cancel{n!})(x-c)}{(2n+1)(\cancel{n!})}\right| = \left|\frac{1}{2}(x-c)\right| < 1$ $|x-c| < 2$

$-2 < x-c < 2$

$-2+c < x < c+2$

$c-2 < x < c+2$

possible I.O.C.

$[(c-2, c+2)]$

test $c-2$:

$\sum_{n=1}^{\infty} \frac{n!(-2)^n}{1\cdot 3\cdot 5\cdots(2n-1)}$ diverges by nth term test

test $c+2$:

$\sum_{n=1}^{\infty} \frac{n!(2)^n}{1\cdot 3\cdot 5\cdots(2n-1)}$ diverges by nth term test

$$\boxed{\begin{array}{c}\text{I.O.C. } (c-2, c+2) \\ c-2 < x < c+2\end{array}}$$

Write an equivalent series

42) $\sum_{n=0}^{\infty}(-1)^{n+1}(n+1)x^n = \boxed{\sum_{n=1}^{\infty}(-1)^n(n)x^{n-1}}$

44) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1} = \boxed{\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^{2n-1}}{2n-1}}$