2017

9.9 Geometric Power Series p.662 #1-36

$$\boxed{* \; S_n = \frac{a_1}{1-r}}$$

Find geometric power series:

2) $f(x) = \frac{1}{2+x} \rightarrow$ convert into $\frac{a_1}{1-r} \rightarrow \frac{\frac{1}{2}}{\frac{2}{2}+\frac{x}{2}} \rightarrow \frac{\frac{1}{2}}{1-(-x/2)}$ $= \sum_{n=0}^{\infty} \frac{1}{2}\left[\frac{-x}{2}\right]^n$

a_1

r

$= \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^{n+1}}$

$\lvert r\rvert < 1$	$\lvert x-0\rvert < 2$	$\boxed{\text{I.O.C. } (-2,2) \quad -2<x<2}$
$\lvert \frac{-x}{2}\rvert < 1$	$-2<x<2$	
$\lvert x\rvert < 2$	* Geometric power series, so endpts diverge	

2b) Long division

$$\begin{array}{r}
\frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} \\
2+x \;\big)\; 1 \qquad\qquad\qquad \\
\ominus\; 1 \mathbin{\overset{\ominus}{+}} \frac{x}{2} \qquad\qquad \\
\hline
-\frac{x}{2} \qquad\qquad \\
\oplus \frac{x}{2} \oplus \frac{x^2}{4} \qquad \\
\hline
\frac{x^2}{4} \qquad \\
\ominus\; \frac{x^2}{4} \mathbin{\overset{\ominus}{+}} \frac{x^3}{8} \\
\hline
\end{array}$$

. . . .

4) $f(x)=\dfrac{2}{5-x} \rightarrow \dfrac{a_1}{1-r} \rightarrow \dfrac{\frac{2}{5}}{\frac{5}{5}-\frac{x}{5}} = \dfrac{\frac{2}{5}}{1-\frac{x}{5}}$ — a_1, r

$$\sum_{n=0}^{\infty} \frac{2}{5}\left[\frac{x}{5}\right]^n = \sum_{n=0}^{\infty} \frac{2x^n}{5^{n+1}}$$

$|r|<1$

$\left|\frac{x}{5}\right|<1$

$|x|<5$

$|x-0|<5$

$-5<x<5$

$\boxed{\text{I.O.C. } (-5,5) \quad -5<x<5}$

$$5-x \overline{)\,2\,} \quad \frac{2}{5}+\frac{2x}{25}+\frac{2x^2}{125}+\dots$$

$\ominus 2 \oplus \frac{2}{5}x$

$\frac{2}{5}x$

$\ominus \frac{2}{5}x \oplus + \frac{2x^2}{25}$

$+\frac{2x^2}{25}$

Find Power Series centered at c

6) $f(x)=\dfrac{2}{6-x} \quad c=-2$

$f(x)=\dfrac{2}{6-(x+2)+2}$

$f(x)=\dfrac{2}{8-(x+2)} \rightarrow \dfrac{a_1}{1-r} \rightarrow \dfrac{\frac{2}{8}}{\frac{8}{8}-\left[\frac{x+2}{8}\right]} = \dfrac{1/4}{1-\left[\frac{x+2}{8}\right]}$ — a_1, r

$$\sum_{n=0}^{\infty} \frac{1}{4}\left[\frac{x+2}{8}\right]^n = \sum_{n=0}^{\infty} \frac{(x+2)^n}{2^{2+3n}}$$

$|r|<1$

$\left|\frac{x+2}{8}\right|<1$

$|x+2|<8$

$|x-c|<r$

$-8<x+2<8$

$-10<x<6$

$\boxed{\text{I.O.C } (-10,6) \quad -10<x<6}$

---

8) $f(x)=\dfrac{1}{1-5x} \quad c=0 \rightarrow \dfrac{1}{1-5x}$ — a_1, r

$$\sum_{n=0}^{\infty}(5x)^n$$

$|r|<1$

$|5x|<1$

$|x|<1/5$

$-1/5<x<1/5$

$\boxed{\text{I.O.C}: (-1/5, 1/5) \quad -1/5<x<1/5}$

10) $f(x) = \dfrac{3}{2x-1}$ $c=2$ $\dfrac{3}{-1+2x} = \dfrac{3}{-1+2(x-2)+4}$

$= \dfrac{3}{3+2(x-2)} = \dfrac{\frac{3}{3}}{\frac{3}{3} - \left[-\frac{2(x-2)}{3}\right]} = \dfrac{1 \leftarrow a_1}{1 - \left[\frac{-2(x-2)}{3}\right]_{\leftarrow r}}$ $\displaystyle\sum_{n=0}^{\infty} \left[\frac{-2(x-2)}{3}\right]^n$

$|r| < 1$

$\left|\dfrac{-2(x-2)}{3}\right| < 1$

$|x-2| < \frac{3}{2}$

$-\frac{3}{2} < x-2 < 3/2$

$-\frac{3}{2} + 2 < x < 2 + 3/2$

$\frac{1}{2} < x < 7/2$

$\boxed{\text{I.O.C. } (1/2, 7/2) \quad \frac{1}{2} < x < 7/2}$

$\displaystyle\sum_{n=0} \frac{(-2)^n (x-2)^n}{3^n}$

---

12) $f(x) = \dfrac{4}{3x+2}$, $c=3$ $\dfrac{4}{2+3x} = \dfrac{4}{2+3(x-3)+9}$

$= \dfrac{4}{11+3(x-3)} = \dfrac{\frac{4}{11}}{\frac{11}{11} - \left[-\frac{3(x-3)}{11}\right]} = \dfrac{\frac{4}{11}}{1 - \left[\frac{-3(x-3)}{11}\right]} = \displaystyle\sum_{n=0} \frac{4}{11}\left[\frac{-3(x-3)}{11}\right]^n$

$|r| < 1$

$\left|\dfrac{-3(x-3)}{11}\right| < 1$

$|x-3| < \frac{11}{3}$

$-\frac{11}{3} < x-3 < \frac{11}{3}$

$3 - \frac{11}{3} < x < 3 + \frac{11}{3}$

$\boxed{\frac{-2}{3} < x < \frac{20}{3} \quad \text{I.O.C. } \left(\frac{-2}{3}, \frac{20}{3}\right)}$

$= 4 \displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n (x-3)^n}{11^{n+1}}$

$A = 3 \quad B = 2$

14) $g(x) = \dfrac{3x-8}{3x^2+5x-2}$, $c = 0$ $\quad \dfrac{A}{3x-1} + \dfrac{B}{x+2}$

$(3x-1)(x+2) \quad x = 1/3 \quad x = -2$

$\dfrac{-3}{3x-1} = \dfrac{3}{1-3x} = \sum_{n=0}^{\infty} 3(3x)^n \quad |3x| < 1 \quad \boxed{-1/3 < x < 1/3}$

$\dfrac{2}{x+2} = \dfrac{2}{2+x} = \dfrac{\frac{2}{2}}{\frac{2}{2}+\frac{x}{2}} = \dfrac{1}{1-\left[\frac{-x}{2}\right]} \quad \sum_{n=0}^{\infty} \left(\frac{-x}{2}\right)^n \quad \left|\frac{x}{2}\right| < 1 \quad \boxed{-2 < x < 2}$

$|x| < 2$

Look for overlap between the series to find I.O.C. $\frac{-1}{3} < x < 1/3$ or $(-1/3, 1/3)$

16) $f(x) = \dfrac{5}{5+x^2}$, $c = 0$ $\quad \dfrac{\frac{5}{5}}{\frac{5}{5} - \left(\frac{-x^2}{5}\right)} = \dfrac{1}{1-\left[\frac{-x^2}{5}\right]} \quad \sum_{n=0}^{\infty} \left(\frac{-x^2}{5}\right)^n$

$|r| < 1$

$\left|\frac{-x^2}{5}\right| < 1$

$|-x^2| < 5$

$|x| < \sqrt{5}$

$-\sqrt{5} < x < \sqrt{5}$

$\boxed{\text{I.O.C. } (-\sqrt{5}, \sqrt{5}) \quad -\sqrt{5} < x < \sqrt{5}}$

18) $h(x) = \dfrac{x}{x^2-1} = \dfrac{1}{2(1+x)} - \dfrac{1}{2(1-x)}$

$\dfrac{1}{2+x} = \dfrac{1/2}{\frac{2}{2} + x/2} = \dfrac{1/2}{1 + x/2} = \sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{x}{2}\right)^n$

$\dfrac{1}{2(1-x)} = \dfrac{1}{2-2x} = \dfrac{1/2}{\frac{2}{2} - \frac{2x}{2}} = \dfrac{\frac{1}{2}}{1-\frac{x}{2}} = \sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{-x}{2}\right)^n$

$\frac{1}{2} \sum \left(\frac{x}{2}\right)^n - \left(\frac{-x}{2}\right)^n = \frac{1}{2} \sum (-2)(x^{2n+1})$ $\quad \boxed{\text{I.O.C. } (-1, 1)}$

20) $f(x) = \frac{2}{(x+1)^3} = \frac{d^2}{dx^2}\left[\frac{1}{x+1}\right]$

$$\frac{1}{x+1} = \frac{1}{1-(-x)} = \sum(-x)^n = \sum(-1)^n x^n \rightarrow$$

$$\frac{d}{dx}\left[\sum(-1)^n x^n\right] = \sum(-1)^n n \cdot x^{n-1}$$

$$\frac{d}{dx}\left[\sum(-1)^n n x^{n-1}\right] = \sum(-1)^n \cdot n(n-1)\, x^{n-2} = \sum_{n=0}^{\infty}(-1)^n n(n-1)x^n \quad |x|<1$$

$\boxed{I.O.C\ (-1,1)}$

22) $f(x) = \ln(1-x^2) = \int \frac{1}{1+x}dx - \int \frac{1}{1-x}dx$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty}(-x)^n \rightarrow \int \sum_{n=0}^{\infty}(-1)x^n dx = \frac{(-1)^n x^{n+1}}{n+1} + C_1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty}(x)^n \rightarrow \int \sum_{n=0}^{\infty} x^n = \frac{x^{n+1}}{n+1} + C_2$$

$$\underbrace{\frac{(-1)^n x^{n+1}}{n+1} + C_1 - \left[\frac{x^{n+1}}{n+1} + C_2\right]} = C + \sum \frac{(-1)x^{2n+2}}{2n+2} \quad |x^2|<1$$

odd powered terms will cancel
and even-powered terms will duplicate (double)
(negative)

$\boxed{IOC\ (-1,1)}$

24) $f(x) = \ln(x^2+1)$

$$f'(x) = \frac{2x}{1+x^2} = \sum(-1)^n 2x\,(x^2)^n$$

$$= \frac{2x}{1-(-x^2)}$$

$$\int \sum(-1)^n 2x(x^{2n}) = \int \sum(-1)^n 2x^{2n+1}dx$$

$$= \frac{2x^{2n+2}}{2n+2} = \frac{\cancel{2}x^{2n+2}}{\cancel{2}(n+1)} \quad \sum_{n=0}^{\infty}\frac{x^{2n+2}}{n+1}(-1)^n \quad |x^2|<1 \quad [(-1,1)]$$

* Endpts converge by Alt. series test

$\boxed{I.O.C : [-1,1]}$

26) $f(x) = \arctan 2x$

$$f'(x) = \frac{2}{1+(2x)^2} = \frac{2}{1+4x^2} = \sum 2(-4x^2)^n = 2\sum (-1)^n 4^n x^{2n}$$

$$= \frac{2}{1-(-4x^2)} \qquad\qquad = 2\sum_{n=0}^{\infty} (-1)^n (2x)^{2n}$$

$$\int f'(x)\,dx = \int 2\sum (-1)^n (2x)^{2n}\,dx = 2\sum (-1)^n \frac{(2x)^{2n+1}}{2n+1} \qquad |2x| < 1 \quad |x| < 1/2$$

$$-1/2 < x < 1/2$$

Test endpts

$\underline{x = -1/2}$: diverges by comparison test with harmonic series

$\underline{x = 1/2}$: converges by Alt. series test

$\boxed{\text{I.O.C. } (-1/2, 1/2]}$

30)

32) Use series $f(x) = \arctan x$ to approximate value, using $R_N \leq 0.001$

$$\int_0^{3/4} \arctan x^2 \, dx \qquad \arctan x^2 = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{4n+2}}{2n+1}$$

34)

36) $\frac{x}{(1-x)^2}$ ... since $\frac{1}{(1-x)^2} = \frac{d}{dx}\left[\frac{1}{1-x}\right] = \frac{d}{dx}\left[\sum x^n\right] = \sum_{n=0}^{\infty} n \cdot x^{n-1}$

So $x \cdot \frac{1}{((1-x)^2} = x \cdot \sum_{n=0}^{\infty} n x^{n-1} = \sum n \cdot x^1 \cdot x^{n-1} = \boxed{\sum_{n=0}^{\infty} n x^n \quad |x| < 1}$