Calculus BC **Ch. 9.1-9.3 Review WS 2**

a) Determine if **Sequence** converge/diverge b) Determine of the **Series** converge/diverge/inconclusive (State the appropriate test to justify answer) c) Find the sum of the **series** if possible (Geometric or Telescoping)

1. $$\sum_{n=1}^{\infty} \frac{3}{4^n}$$

2. $$\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^2}$$

3. $$\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^{3n}$$

4. $$\sum_{n=1}^{\infty} 9^{1-n} 8^{n+1}$$

5. $$4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \ldots.$$

6. $$\sum_{n=1}^{\infty} \frac{3}{n^2 + 3n}$$

7. $\displaystyle\sum_{n=1}^{\infty} 3\left(\frac{5}{2}\right)^{n}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{12-9n-n^{2}}{2n^{2}+1}$

9. $\displaystyle\sum_{n=1}^{\infty} n^{-2/3}$

10. $\displaystyle\sum_{n=1}^{\infty} e^{n-3}2^{4-n}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n+1}}$

13. $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^{n}}{3^{n}}$

Calculus BC   Ch. 9.1-9.3 Review WS 2   Key

a) Determine if **Sequence** converge/diverge b) Determine of the **Series** converge/diverge/inconclusive (State the appropriate test to justify answer) c) Find the sum of the **series** if possible (Geometric or Telescoping)

1. $\sum_{n=1}^{\infty} \frac{3}{4^n}$

a) $\lim_{n\to\infty} 3\left(\frac{1}{4}\right)^n = 0$ (converges)

(Geometric Series Test)

b) $3\sum\left(\frac{1}{4}\right)^n$ By GST, $|r| = \left|\frac{1}{4}\right| < 1$ series converge

c) $\text{Sum} = \frac{a_1}{1-r} = \frac{3/4}{1-1/4} = \frac{3/4}{3/4} = \boxed{1}$

2. $\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^2}$

a) Sequence converges to 0 $\lim_{n\to\infty} \frac{2}{n(\ln n)^2} = 0$

b) $\frac{2}{x(\ln x)^2}$ is positive, decreasing, continuous

$\int \frac{2}{x(\ln x)^2} dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$2\int \frac{1}{x \cdot u^2} x\,du = 2\int_2^{\infty} u^{-2} du = \left.\frac{2u^{-1}}{-1}\right]_2^b$

$= \lim_{b\to\infty} \left.\frac{-2}{\ln x}\right]_2^b = \lim_{b\to\infty} \frac{-2}{\ln b} - \left(\frac{-2}{\ln 2}\right) = 0 + \frac{2}{\ln 2}$

Converges by Integral Test.

3. $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^{3n} = \sum\left(\frac{1}{4^3}\right)^n = \sum\left(\frac{1}{64}\right)^n$

a) $\lim_{n\to\infty} \frac{1}{4^{3n}} = 0$, sequence converges.

b) By GST $|r| = \left|\frac{1}{64}\right| < 1$, series converge

c) $\text{Sum} = \frac{a_1}{1-r} = \frac{\frac{1}{64}}{1-\frac{1}{64}} = \frac{\frac{1}{64}}{\frac{63}{64}}$

$= \frac{1}{\cancel{64}} \cdot \frac{\cancel{64}}{63}$ $\boxed{\text{Sum} = \frac{1}{63}}$

4. $\sum_{n=1}^{\infty} 9^{1-n} 8^{n+1}$ $\sum 9 \cdot 9^{-n} \cdot 8^n \cdot 8$

$= 72\sum \frac{8^n}{9^n} = 72\sum\left(\frac{8}{9}\right)^n$

a) $\lim_{n\to\infty} 72\sum\left(\frac{8}{9}\right)^n = 0$, sequence converges to 0.

b) By GST $|r| = \left|\frac{8}{9}\right| < 1$, series converge

c) $\text{Sum} = \frac{72(8/9)}{1-8/9} = \frac{64}{1/9} = 64 \cdot 9 = \boxed{576}$

5. $4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \ldots$

$\sum_{n=0}^{\infty} 4\left(\frac{-1}{3}\right)^n$ a) $\lim_{n\to\infty} 4\left(\frac{-1}{3}\right)^n = 0$ sequence converges

b) By GST, $|r| = \left|\frac{-1}{3}\right| < 1$, series converge.

c) $\text{Sum} = \frac{a_1}{1-r} = \frac{4}{1-(-1/3)} = \frac{4}{4/3} = \boxed{3}$

6. $\sum_{n=1}^{\infty} \frac{3}{n^2+3n} = \frac{A}{n} + \frac{B}{n+3}$

$A = 1$ $B = -1$

$n = 0$ $n = -3$

a) $\lim_{n\to\infty} \frac{3}{n^2+3n} = 0$ sequence converges to 0.

$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+3}$

$\left(\frac{1}{1} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{5}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right) + \left(\cancel{\frac{1}{5}} - \frac{1}{8}\right) + \left(\cancel{\frac{1}{6}} - \frac{1}{9}\right)\ldots$

$= 1 + \frac{1}{2} + \frac{1}{3}$

$= \frac{6}{6} + \frac{3}{6} + \frac{2}{6} = \frac{11}{6}$

$\boxed{\text{Series converges to } \frac{11}{6} \text{ by Telescoping Series}}$

7. $\sum_{n=1}^{\infty} 3\left(\frac{5}{2}\right)^n$ a) $\lim_{n\to\infty} 3\left(\frac{5}{2}\right)^n = \infty$, sequence diverges

b) By GST since $r = \left|\frac{5}{2}\right| > 1$, series diverge (or n^{th} term test, diverges)

8. $\sum_{n=1}^{\infty} \frac{12-9n-n^2}{2n^2+1}$ a) $\lim_{n\to\infty} \frac{12-9n-1n^2}{2n^2+1} = \frac{-1}{2}$ (sequence converges)

b) Diverges by n^{th} term test since $\lim_{n\to\infty} a_n = -\frac{1}{2} \neq 0$

9. $\sum_{n=1}^{\infty} n^{-2/3}$ $\quad \sum \frac{1}{n^{2/3}}$ a) sequence converges to 0

b) By p-series test, $p = \frac{2}{3} < 1$, series diverge.

10. $\sum_{n=1}^{\infty} e^{n-3} 2^{4-n}$ $\quad \sum e^n \cdot e^{-3} \cdot 2^4 \cdot 2^{-n}$

$\frac{16}{e^3} \sum \frac{e^n}{2^n} = \frac{16}{e^3} \sum \left(\frac{e}{2}\right)^n$ a) sequence diverges to ∞

b) By GST, series diverge $|r| = \left|\frac{e}{2}\right| > 1$

11. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ a) $\lim_{n\to\infty} \frac{\ln n}{n} = 0$ (sequence converges to 0)

b) n^{th} term test inconclusive

$\frac{\ln x}{x}$ is positive, decreasing, continuous

$\lim_{b\to\infty} \int_1^b \frac{\ln x}{x}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\lim_{b\to\infty} \int_1^b \frac{u \cdot \cancel{x}\,du}{\cancel{x}} = \frac{u^2}{2}\Big]_1^b$

$\lim_{b\to\infty} \frac{b^2}{2} - \frac{1}{2} = \infty$

Series Diverges by Integral Test.

12. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n+1}}$ a) $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n+1}} = 0$ Sequence converges to 0.

b) $\frac{1}{\sqrt[3]{x+1}}$ is positive, decreasing, continuous.

$\lim_{b\to\infty} \int_1^b \frac{1}{\sqrt[3]{x+1}}\,dx$ $\quad u = x+1$, $\frac{du}{dx} = 1$, $dx = du$ $\quad \int \frac{1}{u^{1/3}}\,du = \int u^{-1/3}\,du$

$\lim_{b\to\infty} \frac{(x+1)^{2/3}}{2/3}\Big]_1^b = \lim_{b\to\infty} \frac{3}{2}(b+1)^{2/3} - \frac{3}{2}(2)^{2/3} = \infty$

Series diverge by Integral Test

14 a) $\lim_{n\to\infty} a_n = 0$ (sequence diverges to 0)

13. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$ $\quad$ *Definition of e $\quad \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$

a) sequence converges to e

b) Series diverge by n^{th} term test since $\lim_{n\to\infty} a_n \neq 0$

$\boxed{\text{OR}}$

$y = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$

$\ln y = \lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right)^n$

$\ln y = \lim_{n\to\infty} n \cdot \ln\left(1+\frac{1}{n}\right)$

$\ln y = \lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}$

L'Hopital's

$\ln y = \lim_{n\to\infty} \frac{\frac{-1n^{-2}}{1+\frac{1}{n}}}{-1n^{-2}} = \frac{1}{1+\frac{1}{n}} = 1$

$\ln y = 1$

$\log_e y = 1$ $\quad e^1 = y$ $\quad y = \boxed{e}$

14. $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$ $\quad \sum \frac{1}{3^n} + \frac{2^n}{3^n} = \sum\left(\frac{1}{3}\right)^n + \sum\left(\frac{2}{3}\right)^n$

b) By GST series converge since $r_1 = \left|\frac{1}{3}\right| < 1$, $r_2 = \left|\frac{2}{3}\right| < 1$

c) Sum $= \frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{\frac{2}{3}}{1-\frac{2}{3}}$

$= \frac{\frac{1}{3}}{\frac{2}{3}} + \frac{\frac{2}{3}}{\frac{1}{3}} =$

$= \frac{1}{2} + 2 = \boxed{2.5 \text{ or } \frac{5}{2}}$