8.5 Homework - Arc Length

p. 618-620 #9, 17, 23, 26, 29, 31, 36, 42, 47

Find Arc Length: $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

9) $y = x^{3/2}$ from $x = 0$ to $x = 4$

$y' = \frac{3}{2}x^{1/2}$ $\quad L = \int_0^4 \sqrt{1 + \left[\frac{3}{2}\sqrt{x}\right]^2}\, dx \to \sqrt{\frac{4}{4} + \frac{9x}{4}}$

$\frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$ $\quad u = 4+9x$, $\frac{du}{dx} = 9$ $\quad \Big|\ dx = \frac{du}{9}$ $\quad \Big|\ \frac{1}{2}\int u^{1/2} \cdot \frac{du}{9}$

$\frac{1}{18}\int u^{1/2} du \to \frac{1}{18} \cdot \frac{u^{3/2}}{3/2} \to \frac{1}{18} \cdot \frac{2}{3} u^{3/2} \to \frac{1}{27}(4+9x)^{3/2}\Big]_0^4 \to \frac{1}{27}\left[(40)^{3/2} - 4^{3/2}\right]$

$\frac{1}{27}\left[40\sqrt{40} - 8\right] \to \boxed{\frac{1}{27}\left[80\sqrt{10} - 8\right]}$

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17) $8y = x^4 + \frac{2}{x^2}$ from $[1, 2]$

$y = \frac{x^4}{8} + \frac{2}{8x^2}$ $\quad y' = \frac{1}{8} \cdot 4x^3 + \frac{1}{4} \cdot -2x^{-3}$

$y = \frac{1}{8}x^4 + \frac{1}{4}x^{-2}$ $\quad y' = \frac{1}{2}x^3 - \frac{1}{2x^3}$

$L = \int_1^2 \sqrt{1 + \left[\frac{x^3}{2} - \frac{1}{2x^3}\right]^2}\, dx \to \sqrt{1 + \frac{x^6}{4} - \frac{1}{2} + \frac{1}{4x^6}}\, dx \to \sqrt{\frac{x^6}{4} + \frac{1}{2} + \frac{1}{4x^6}}$

$\int_1^2 \sqrt{\left(\frac{x^3}{2} + \frac{1}{2x^3}\right)^2}\, dx \to \frac{x^4}{8} - \frac{1}{4x^2}\Big]_1^2 = \boxed{\frac{33}{16}}$

$\boxed{8.5}$ Find arc Length by partitioning y-axis

23) $y = x^{2/3}$ $[0, 1]$ $L = \int_{y_1}^{y_2} \sqrt{1+[f'(y)]^2}\,dy$

$x = y^{3/2}$ $\overset{y_1\ \ y_2}{[0, 1]}$ $f'(y) = \frac{3}{2}y^{1/2}$

$L = \int_0^1 \sqrt{1+\left[\frac{3}{2}\sqrt{y}\right]^2}\,dy \rightarrow \sqrt{1+\frac{9}{4}y} \rightarrow \sqrt{\frac{4}{4}+\frac{9}{4}y} \rightarrow \frac{1}{2}\int_0^1 \sqrt{4+9y}\,dy$

$u = 4+9y$

$\frac{du}{dy} = 9$ $\frac{1}{2}\int u^{1/2}\cdot\frac{du}{9}$ $\rightarrow \boxed{L = \frac{1}{27}\left[13\sqrt{13}-8\right]}$

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26) $x = \frac{2}{3}(y-5)^{3/2}$ from $y=5$ to $y=6$ $x' = \frac{2}{3}\cdot\frac{3}{2}(y-5)^{1/2}$

$L = \int_5^6 \sqrt{1+\left[[y-5]^{1/2}\right]^2}\,dy \rightarrow \sqrt{1+y-5} \rightarrow \int_5^6 \sqrt{y-4}\,dy = \boxed{\frac{2}{3}\left[2\sqrt{2}-1\right]}$

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29) $y = \sqrt{25-x^2}$ $[0, 4]$ $y = (25-x^2)^{1/2}$

$y' = \frac{1}{2}(25-x^2)^{-1/2}(-2x) \rightarrow \frac{-x}{\sqrt{25-x^2}}$

$L = \int_0^4 \sqrt{1+\left[\frac{-x}{\sqrt{25-x^2}}\right]^{1/2}}\,dx$

$= \int_0^4 \sqrt{\frac{25-x^2+x^2}{25-x^2}} \rightarrow \int_0^4 \sqrt{\frac{25}{25-x^2}}\,dx \approx \boxed{4.636}$

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31) $y = \sin x$ $[0, \pi/2]$ $y' = \cos x$

$L = \int_0^{\pi/2} \sqrt{1+\cos^2 x}\,dx \approx \boxed{1.910}$

36) Find Arc Length $F(x) = \int_0^{4x} \sqrt{t^4 - \frac{1}{4}}\, dt$ $[1, 2]$

$F'(x) = \sqrt{(4x)^4 - \frac{1}{4}} \cdot 4$ ← SFTC

$F'(x) = 4\sqrt{1024x^4 - \frac{1}{4}}$

$L = \int_1^2 \sqrt{1 + \left[4\sqrt{1024x^4 - 1/4}\right]^2}\, dx \approx \boxed{149.321}$

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42) $y = \frac{1}{2}(e^x + e^{-x})$ $[0, 2]$

$y' = \frac{1}{2}(e^x - e^{-x})$ $L = \int_0^2 \sqrt{1 + \left[\frac{e^x - e^{-x}}{2}\right]^2}\, dx \approx \boxed{3.627}$

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47) $y = 5x - x^2$ above x-axis

$5x - x^2 = 0$
$x(5-x) = 0$ | $x = 0, 5$

$y' = 5 - 2x$

$L = \int_0^5 \sqrt{1 + (5-2x)^2}\, dx \approx \boxed{13.904}$