BC Calculus – 10.1 Notes – Convergent & Divergent Infinite Series

Recall: Writing terms of a sequence.

$$a_n = \{1 + (-2)^n\}$$

$$-1, 5, -7, 17, -31$$

Sequence: A collection of numbers that are in one-to-one correspondence with positive integers.

$$-2 \qquad 4 \qquad -\frac{26}{6} \qquad \frac{80}{24} \qquad -\frac{242}{120}$$

Monotonic Sequences never decreases or never increases	**Bounded Sequences**
$a_1 \leq a_2 \leq a_3 \leq \cdots \leq a_n$ or $a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_n$	$a_n \leq M$ (upper bound / above) $a_n \geq N$ (lower bound / below) $\{a_n\}$ bounded if both are true

Infinite Series:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n$$

Partial Sum:

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

a_n vs S_n:

a_n is an expression that gives the

S_n is an expression that gives the

1. Use the following sequence $2, 4, 6, 8, 10$ to find a_4 and S_4.

$$\sum_{n=1}^{\infty} a_n =$$

②

Convergent and Divergent Series

For the infinite series $\sum_{n=1}^{\infty} a_n$, the n^{th} partial sum is $S_n = a_1 + a_2 + a_3 + \cdots + a_n$.

If the sequence of the partial sum $\{S_n\}$ to S, then the series $\sum_{n=1}^{\infty} a_n$ The limit S is called the sum of the series.

Likewise, if $\{S_n\}$ then the series

2. **Does the series converge or diverge?** $\sum_{n=1}^{\infty} \frac{1}{2^n}$

3. Use a calculator to find the partial sum S_n of the series $\sum_{n=1}^{\infty} \frac{10}{n(n+2)}$ for $n = 200, 1000$.

4. **Does the series converge or diverge?** $\sum_{n=1}^{\infty} n$

10.1 Convergent and Divergent Infinite Series

Calculus

Practice

1. Given the infinite series $\sum_{n=1}^{\infty} (-1)^n$, find the sequence of partial sums $S_1, S_2, S_3, S_4,$ and S_5.

2. Find the sequence of partial sums $S_1, S_2, S_3, S_4,$ and S_5 for the infinite series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \cdots$.

3. If the infinite series $\sum_{n=1}^{\infty} a^n$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$, what is the sum of the series?

4. The infinite series $\sum_{n=1}^{\infty} a^n$ has nth partial sum $S_n = \frac{n}{4n+1}$ for $n \geq 1$. What is the sum of the series?

5. Use a calculator to find the partial sum S_n of the series $\sum_{n=1}^{\infty} \frac{6}{n(n+3)}$ for $n = 100, 500, 1000$.

6. Show that the sequence with the given nth term $a_n = 1 + 2n$ is monotonic.

7. What is the nth partial sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{2^{n+1}}$?

8. Which of the following could be the nth partial sum for the infinite series $\sum_{n=1}^{\infty} \frac{1}{4^n}$?

(A) $S_n = \frac{1}{3}\left(1 + \frac{1}{4^n}\right)$ (B) $S_n = \frac{1}{3}\left(1 - \frac{1}{4^{n+1}}\right)$ (C) $S_n = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$ (D) $S_n = \frac{1}{4}\left(1 - \frac{1}{3^n}\right)$

9. If the infinite series $\sum_{n=1}^{\infty} a_n$ is convergent and has a sum of $\frac{7}{8}$, which of the following could be the nth partial sum?

(A) $S_n = \frac{7n+1}{8n^2+1}$ (B) $S_n = \frac{7n^2+1}{8n+1}$

(C) $S_n = 2\left(\frac{7}{8} - \frac{1}{n+2} - \frac{1}{n+3}\right)$ (D) $S_n = \left(\frac{7}{8} - \frac{1}{n+2} - \frac{1}{n+3}\right)$

10. Which of the following sequences with the given nth term is bounded and monotonic?

(A) $a_n = 2 + (-1)^n$ (B) $a_n = \frac{n^2}{n+1}$ (C) $a_n = \frac{3n}{n+2}$ (D) $a_n = \frac{\cos n}{n}$