10.3 AP Practice Problems (p. 746) - Tests for Convergence (nth term, p-series, Integral Test)

1. For what numbers p does the series $\sum_{k=1}^{\infty} \frac{1}{k^{p/3}}$ converge?

(A) $p > 1$ (B) $|p| > 1$ (C) $p > 3$ (D) $p > \frac{1}{3}$

2. Which of the following series diverge?

I. $\sum_{k=1}^{\infty} \frac{e^{k-1}}{3^{k-1}}$ II. $\sum_{k=1}^{\infty} \cos\left(\pi + \frac{1}{k}\right)$ III. $\sum_{k=1}^{\infty} \frac{10}{k}$

(A) I and II only (B) I and III only
(C) II and III only (D) I, II, and III

3. For what values of p does the series $\sum_{k=1}^{\infty} k^p$ converge?

(A) $-1 < p < 1$ (B) $p < -1$
(C) $p > 1$ (D) The series diverges.

4. $\sum_{k=1}^{\infty} \frac{5^{k-1} - 3^{k-1}}{8^{k-1}} =$

(A) $\frac{1}{3}$ (B) $\frac{16}{15}$ (C) $\frac{49}{24}$ (D) $\frac{34}{15}$

(26)

5. Determine whether the series $\sum_{k=1}^{\infty} \frac{2}{k^{3/2}}$ converges or diverges. If it converges, find bounds for the sum of the series.

(A) Converges; $\frac{3}{2} < \sum_{k=1}^{\infty} \frac{2}{k^{3/2}} < \frac{5}{2}$

(B) Converges; $2 < \sum_{k=1}^{\infty} \frac{2}{k^{3/2}} < 3$

(C) Converges; $4 < \sum_{k=1}^{\infty} \frac{2}{k^{3/2}} < 6$

(D) The series diverges.

6. (a) Given the infinite series $\sum_{k=3}^{\infty} \frac{\ln k}{k}$, find a function f with the property that $f(k) = a_k$ for all positive integers $k \geq 3$.

(b) Show that f is continuous, positive, and decreasing on the interval $[3, \infty)$.

(c) Determine whether the series $\sum_{k=3}^{\infty} \frac{\ln k}{k}$ converges or diverges.

7. (a) Show that the infinite series $\sum_{k=1}^{\infty} \frac{1}{1+9k^2}$ converges.

(b) Find bounds for the sum of the series $\sum_{k=1}^{\infty} \frac{1}{1+9k^2}$.