BC Calculus – 10.3 Notes – *n*th term, p-series, and Integral Test

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n$$

If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n =$

If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$

Nth Term Test for Divergence (10.3a)

If $\lim_{n\to\infty} a_n \neq 0$, then

Use the Nth term test to make a conclusion about divergence for each series.

1. $\sum_{n=1}^{\infty} \frac{3n^3+1}{5n^3-2n^2+1}$

2. $\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n$

3. $\sum_{n=1}^{\infty} \frac{1}{n}$

4. $\sum_{n=1}^{\infty} \frac{2^{n+2}}{2^{n+3}+1}$

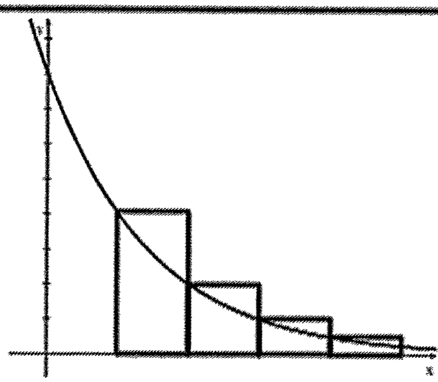
5. $\sum_{n=1}^{\infty} \frac{e^{4n}}{3n}$

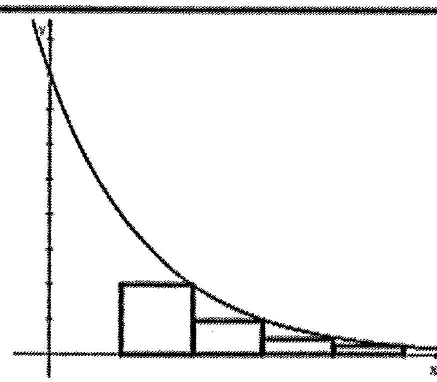
10.3b

Integral Test for Convergence

If f is a positive, continuous, and decreasing function for $x \geq k$, and $a_n = f(x)$, then

and





Determine the convergence or divergence of the series

1. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

2. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

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10.3c

p-Series

Let p be a positive constant of the series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$

The series converges if

The series diverges if

Harmonic Series

Do the following series converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

For what values of *k* will the series converge?

3. $\sum_{n=1}^{\infty} \frac{1}{n^{2k-5}}$

4. $\sum_{n=1}^{\infty} \frac{1}{n(n^{2k})}$

5. $\sum_{n=1}^{\infty} \frac{n}{n^{4k}+5}$

Things we should now recognize

Series

- Geometric
- Harmonic
- *p*-Series

Tests for convergence/divergence

- Nth Term Test for Divergence
- Integral Test

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10.3 The nth Term Test for Divergence (10.3a)

Calculus

Practice

For each of the following series, determine the convergence or divergence of the given series. State the reasoning behind your answer.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{3-2n}{5n+1}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{\sqrt{n^2+1}}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{n+1}}{\pi^n}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{7^n+1}{7^{n+1}}$

6. $\displaystyle\sum_{n=0}^{\infty} 5\left(\frac{5}{2}\right)^n$

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10.3 The nth Term Test for Divergence (10.3a) **Test Prep**

7. The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \sin 2n$ II. $\sum_{n=1}^{\infty} \left(2+\frac{3}{n}\right)$ III. $\sum_{n=1}^{\infty} \frac{n^3+1}{n^2}$

(A) II only (B) III only (C) I and II only (D) I, II, and III

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8. The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \ln\left(\frac{n-1}{n}\right)$ II. $\sum_{n=1}^{\infty} \frac{3n-2n^2}{5n^2}$ III. $\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^n$

(A) II only (B) II and III only (C) I and II only (D) I, II, and III

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9. If $a_n = \cos\left(\frac{\pi}{2n}\right)$ for $n = 1,2,3,\ldots$, which of the following about $\sum_{n=1}^{\infty} a_n$ must be true?

(A) The series converges and $\lim_{n\to\infty} a_n = 0$.

(B) The series diverges and $\lim_{n\to\infty} a_n = 0$

(C) The series diverges and $\lim_{n\to\infty} a_n \neq 0$

(D) The series converges and $\lim_{n\to\infty} a_n \neq 0$

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10.3b **Integral Test for Convergence**

Calculus

Practice

If the Integral Test applies, use it to determine whether the series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{n}{e^n}$

2. $\sum_{n=1}^{\infty} f(n) = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

3. $\sum_{n=1}^{\infty} f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

4. $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin\frac{\pi}{n}$

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5. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

6. $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+2}$, where k is a positive integer. Assume the series meets the criteria for the Integral Test.

7. Let f be a positive, continuous, and decreasing function. If $\int_1^{\infty} f(x)\,dx = 4$, which of the following statements about the series $\sum_{n=1}^{\infty} f(n)$ must be true?

A. $\sum_{n=1}^{\infty} f(n) = 0$

B. $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) > 4$

C. $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) < 4$

D. $\sum_{n=1}^{\infty} f(n)$ diverges, and $\sum_{n=1}^{\infty} f(n) = 0$

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8. Explain why the Integral Test does not apply for the series $\sum_{x=1}^{\infty} e^x \sin x$.

9. Show that the series $\sum_{x=1}^{\infty} \frac{\tan^{-1} x}{x^2+1}$ meets the criteria to apply the Integral Test for convergence.

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10. Let f be positive, continuous, and decreasing on $[1, \infty)$, such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n = 7$, which of the following must be true?

A. $\lim_{n\to\infty} a_n = 7$

B. $\int_1^\infty f(x)\,dx = 7$

C. $\int_1^\infty f(x)\,dx$ diverges

D. $\int_1^\infty f(x)\,dx$ converges

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11. Which of the following can be used to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n+3}{n+4}$?

I. Properties of Geometric Series
II. nth-Term Test
III. Integral Test

A. I only

B. II only

C. III only

D. II and III only

E. I, II, and III

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12. Which of the following can be used to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$?

I. Properties of Geometric Series
II. nth-Term Test
III. Integral Test

A. I only

B. II only

C. III only

D. I and II only

E. I and III only

10.3b Integral Test for Convergence

Test Prep

13. Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. The integral test can be used to determine convergence or divergence of the series because $f(x) = \frac{1}{x^3}$ is positive, continuous, and decreasing on $[1, \infty)$. Which of the following is true?

A. $1 + \int_1^\infty \frac{1}{x^3}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^3} < \int_1^\infty \frac{1}{x^3}\,dx$

B. $\int_1^\infty \frac{1}{x^3}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^3} < 1 + \int_1^\infty \frac{1}{x^3}\,dx$

C. $\sum_{n=1}^{\infty} \frac{1}{n^3} < \int_1^\infty \frac{1}{x^3}\,dx < 1 + \int_1^\infty \frac{1}{x^3}\,dx$

D. $\int_1^\infty \frac{1}{x^3}\,dx < 1 + \int_1^\infty \frac{1}{x^3}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^3}$

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10.3c Harmonic Series and p-series

Calculus

Practice

Determine the convergence or divergence of the following p-series.

1. $\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$

2. $\sum_{n=1}^{\infty} \frac{1}{n^{0.13}}$

3. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

What are all the values of p for which...

4. $\sum_{n=1}^{\infty} \frac{2n}{n^p + 2}$ converges?

5. $\sum_{n=1}^{\infty} \frac{1}{n^{3p}}$ diverges?

6. Both series $\sum_{n=1}^{\infty} n^{-5p}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{5}\right)^n$ converge?

7. $\int_{1}^{\infty} \frac{1}{x^{3p+4}}\, dx$ converges?

Find the positive values of p for which the infinite series converge?

8. $\sum_{n=1}^{\infty} \left(\frac{4}{p}\right)^n$

9. $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^p}$

10. $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$

10.3c Harmonic Series and p-series

Test Prep

11. Which of the following infinite series converge?

I. $\sum_{n=1}^{\infty} n^{-\frac{1}{2}}$ II. $\sum_{n=1}^{\infty} \left(\frac{e}{2}\right)^{-n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n^e}$

A. None B. II only C. III only

D. I and II only E. II and III only

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12. Which of the following infinite series converge?

I. $\sum_{n=1}^{\infty} 3^{-n}$ II. $\sum_{n=1}^{\infty} \frac{1}{(3n+1)^3}$ III. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

A. I only B. II only C. III only

D. I and II only E. I and III only

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13. Which of the following infinite series is a divergent p-series?

A. $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ B. $\sum_{n=1}^{\infty} n^{-\frac{1}{2}}$ C. $\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$ D. $\sum_{n=1}^{\infty} n^{\frac{3}{2}}$

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14. Which of the following is not a p-series?

A. $\sum_{n=1}^{\infty} n^{-3}$

B. $\sum_{n=1}^{\infty} \frac{1}{n}$

C. $\sum_{n=1}^{25} \frac{1}{n^{\pi}}$

D. $\sum_{n=1}^{\infty} \frac{1}{\pi^{n}}$

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15. Which of the following is a harmonic series?

A. $\sum_{n=1}^{\infty} \frac{1}{3n}$

B. $\sum_{n=1}^{\infty} \frac{1}{n}$

C. $\sum_{n=1}^{1000} \frac{1}{n}$

D. $\sum_{n=1}^{\infty} \frac{3n^2}{4n^2+1}$

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16. Find the positive values of k for which the series $\sum_{n=3}^{\infty} \frac{1}{(n \ln n)(\ln(\ln n))^k}$ converges.