BC Calculus – 10.4 Notes – Comparison Tests for Convergence

Comparison Test

Let $0 < a_n \leq b_n$ for all n.

If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$

If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$

Determine if the following series converge or diverge.

1. $\sum_{n=1}^{\infty} \frac{1}{3+2^n}$

2. $\sum_{n=1}^{\infty} \frac{1}{4^n-3}$

3. $\sum_{n=1}^{\infty} \frac{1}{7n^2+4}$

Limit Comparison Test

If $a_n > 0,\ b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ (where L is finite and positive), then

$$\sum_{n=1}^{\infty} a_n \text{ and } \sum_{n=1}^{\infty} b_n$$

28

Determine if the following series converge or diverge.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2 - 2}{5n^5 + 3n + 1}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n^2 + 5n + 5}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n + 2}}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 - 7}{2n^5 + n^2 + n + 1}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{n3^n}{4n^3 + 2}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n}}$

10.4 Comparison Tests for Convergence

Calculus

Practice

1. Which of the following statements about convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\ln(n+2)}$ is true?

   (A) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+2)}$ converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$

   (B) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+2)}$ converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

   (C) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+2)}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

   (D) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+2)}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$

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2. Which of the following series converges?

   (A) $\sum_{n=1}^{\infty} \frac{3n}{n^3+2}$

   (B) $\sum_{n=1}^{\infty} \frac{5n}{2n+1}$

   (C) $\sum_{n=1}^{\infty} \frac{7n}{n^2+1}$

   (D) $\sum_{n=1}^{\infty} \frac{5^n}{4^n+1}$

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3. Use the Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{1}{2+5^n}$ converges or diverges.

30

4. Which of the following series can be used with the Limit Comparison Test to determine convergence of the series $\sum_{n=1}^{\infty} \frac{n^3}{n^4+3}$?

(A) $\sum_{n=1}^{\infty} \frac{n}{n+3}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n^3+3}$

(C) $\sum_{n=1}^{\infty} \frac{1}{n}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n^4}$

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5. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ diverges which of the following must be true?

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

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6. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} b_n$ converges which of the following must be true?

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} a_n$ diverges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} a_n$ converges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} a_n$ diverges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} a_n$ converges.

7. Let $a > 0, b > 0$, and $c > 0$. Determine whether the series $\sum_{n=0}^{\infty} \frac{1}{an^2 + bn + c}$ converges or diverges.

8. Determine the convergence or divergence of the series $\sum_{n=2}^{\infty} \frac{1}{6^n + 6}$.

9. For the series $\sum_{n=1}^{\infty} \frac{n3^n}{2n^4 - 2}$, which of the following could be used with the Limit Comparison Test?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^4}$

(B) $\sum_{n=1}^{\infty} \frac{3^n}{n^4}$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(D) $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

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10. Which of the following can be used with the Comparison Test to determine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{2 + \sqrt{n}}$?

(A) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n}$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(D) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

32

11. Which of the following series diverge?

I. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n}$ II. $\sum_{n=1}^{\infty} \frac{1}{n^3 - 27}$ III. $\sum_{n=1}^{\infty} \frac{1}{4^n + 1}$

(A) I only   (B) II only

(C) I and II only   (D) I, II, and III

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12. Consider the series $\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}$, where $p \geq 0$. For what values of p is the series convergent?

13. Determine whether the series $\sum_{n=1}^{\infty} \frac{n-3}{n^3}$ converges or diverges.

14. Consider the series $1 + \frac{1}{5} + \frac{1}{9} + \frac{1}{13} + \cdots = \sum_{n=1}^{\infty} \frac{1}{4n-3}$. Use the Limit Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{4n}$ to determine the convergence of the series.

10.4 Comparison Tests for Convergence

Test Prep

15. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $a_n \leq b_n$, then which of the following must be true?

(A) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges.

(C) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(D) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ converges.

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16. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\lim_{n\to\infty} \frac{a_n}{b_n} = 2$, then which of the following must be true?

I. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ diverges.

II. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ converges.

III. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges.

IV. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(A) I only (B) II only (C) III only

(D) IV only (E) I and II only (F) III and IV only