10.5 AP Practice Problems (p.765) - Alternating Series Test & Absolute Convergence

1. Which of the following series converge?

I.
$$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k^2}$$

II.
$$\sum_{\substack{k=1\\ \infty\\ \infty}}^{\infty} (-1)^k \left(\frac{5}{3}\right)^k$$

$$\mathbf{m}. \sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{k}}$$

- (A) I only
- (B) I and II only
- (C) I and III only
- (D) I, II, and III
- 2. The alternating series $\sum_{k=1}^{\infty} \frac{(-1)^k k}{10^k}$ converges. What is

the maximum error incurred by using the first three nonzero terms to approximate the sum of the series?

- (A) -0.083
- (B) 0.003
- (C) 0.0004
- (D) 0.0826
- 3. What is the fewest number of terms of the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$

that must be added to approximate the sum so that the error is less than or equal to 0.001?

- (A) 7
- (B) 9
- (C) 10
- (D) 11

4. Which of the following series converge conditionally, but not absolutely?



I.
$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3}{k}$$

II.
$$\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{1}{k}\right)^{4/3}$$

III.
$$\sum_{k=0}^{\infty} (-1)^k \left(\frac{3}{4}\right)^k$$

- (A) I only
- (B) I and II only
- (C) I and III only
- (D) I, II, and III
- 5. (a) Write out the first five terms of the series $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$.
 - (b) Show the series $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$ converges.
- (c) How many terms of the series are necessary to approximate the sum S with an error less than or equal to 0.0001?

6. Determine whether the series $\sum_{k=1}^{\infty} \frac{\cos(2k)}{4^k}$ converges absolutely, converges conditionally, or diverges. Show your work.