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BC Calculus – 10.5 Notes – Alternating Series Test & Absolute Convergence

Alternating Series Test

If $a_n > 0$, then the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ and $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converge if **BOTH** of the following conditions are met:

1.

2.

Ways to check if a_n is decreasing.

- Take the 1st derivative and see if it is negative.
- Usually, it is obvious.
- Could manipulate $a_{n+1} \le a_n$

Determine if the following series converge or diverge.

1. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$

2. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n+5}{(n+2)(n+3)}$

3. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n+7)}{n}$

4. $\sum_{n=1}^{\infty}\cos(n\pi)\frac{1}{n}$

5. The following is not an alternating series. Look carefully to see if you can tell why not.

$$\sum_{n=1}^{\infty}\frac{(-1)^n \cos(n\pi)\, n}{n^2+1}$$

Absolute or Conditional Convergence Notes:

Three possibilities with regards to the series $\sum_{n=1}^{\infty} a_n$ dealing with convergence or divergence.

1. **Converges Absolutely.** If $\sum_{n=1}^{\infty} |a_n|$ converges, then the original series $\sum_{n=1}^{\infty} a_n$ also converges.
2. **Converges Conditionally.** If $\sum_{n=1}^{\infty} |a_n|$ diverges, but the original series $\sum_{n=1}^{\infty} a_n$ converges.
3. **Divergent.** Both $\sum_{n=1}^{\infty} |a_n|$ and $\sum_{n=1}^{\infty} a_n$ diverge.

Find if the series converges absolutely, converges conditionally, or is divergent.

1. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$

3. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$

Find the values of x that make the series converge absolutely.

4. $\sum_{n=1}^{\infty} \frac{(-1)^n n(x+4)^n}{6^n}$

5. $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$

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Write your questions and thoughts here!

5. The following is not an alternating series. Look carefully to see if you can tell why not.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)\, n}{n^2+1}$$

10.5 Alternating Series Test

Calculus

Practice

1. Explain why the Alternating Series Test does not apply to the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{n}$.

2. The Alternating Series Test can be used to show convergence of which of the following alternating series?

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ II. $\sum_{n=2}^{\infty} (-1)^{n+1}\left(\frac{n}{n^2+4}\right)$ III. $\sum_{n=1}^{\infty} (-1)^{n}\left(\frac{4n}{5n+3}\right)$

A. I only B. II only C. III only

D. I and II only E. I, II, and III

3. Which of the following series converge?

A. $\sum_{n=1}^{\infty} (-1)^n\left(\frac{1-2n}{n}\right)$

B. $\sum_{n=1}^{\infty} (-1)^n\left(\frac{n+1}{3n}\right)$

C. $\sum_{n=1}^{\infty} (-1)^n\left(\frac{n^3}{2\sqrt{n}}\right)$

D. $\sum_{n=1}^{\infty} (-1)^n\left(\frac{2\sqrt{n}}{n^3}\right)$

Use the Alternating Series Test to show the series are convergent.

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n^2}\right)$

5. $\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{1}{3^n}\right)$

6. **Calculator active.** Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \frac{(-1)^n}{(-1)^n+\sqrt{n}}$

   I. The series is alternating.
   II. $|a_{n+1}| \leq |a_n|$ for $n \geq 2$.
   III. $\lim_{n\to\infty} a_n = 0$

A. I only   B. I and II only   C. I and III only   D. I, II, and III

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7. **Calculator active.** Which of the following statements about the series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$, where $a_n = \frac{2+\cos n}{n^2}$ is true?

A. The series converges by the Alternating Series Test

B. The Alternating Series Test cannot be used because the series is not alternating.

C. The Alternating Series Test cannot be used because $\lim_{n\to\infty} a_n \neq 0$.

D. The Alternating Series Test cannot be used because the terms of a_n are not decreasing.

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8. The Alternating Series Test can be used to show convergence for which of the following series?

A. $\frac{2}{1} - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \frac{6}{5} - \cdots$, where $a_n = \frac{(-1)^{n+1}(n+1)}{n}$.

B. $\frac{2}{1} - \frac{1}{1} + \frac{2}{2} - \frac{1}{2} + \frac{2}{3} - \frac{1}{3} + \frac{2}{4} - \frac{1}{4} + \cdots$

C. $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \cdots$, where $a_n = (-1)^{n+1}\frac{1}{n^2}$

D. $\frac{3}{2} - \frac{2}{2} + \frac{3}{3} - \frac{2}{3} + \frac{3}{4} - \frac{2}{4} + \cdots$

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9. For which of the following series can the Alternating Series Test not be used?

A. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$

B. $\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n^3)}{n}$

C. $\sum_{n=4}^{\infty} \frac{(-1)^n n}{n-3}$

D. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

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10. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ is true?

A. The series diverges by comparison to $\frac{1}{n}$.

B. The series converges by comparison to $\frac{1}{n}$.

C. The series diverges by the Alternating Series Test.

D. The series converges by the Alternating Series Test.

11. Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)!}{(n)!}$?

I. The series is alternating.

II. $|a_{n+1}| \leq |a_n|$ for $n \geq 1$.

III. $\lim_{n\to\infty} a_n = 0$

A. I only B. I and II only C. I and III only D. I, II, and III

10.5 Alternating Series Test

Test Prep

12. The Alternating Series Test can be used to show convergence for which of the following series?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n^2}\right)$

II. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin n}{n^2}$

III. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}+1} - \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}+1} - \frac{1}{\sqrt{4}-1} + \cdots\right)$

A. I only B. I and II only C. II and III only D. I, II, and III

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13. If $\sum_{n=1}^{\infty} \frac{(-1)^n}{a_n}$ converges, which of the following must be true?

A. $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \geq a_n > 0$ for $n \geq 1$.

B. $\lim_{n\to\infty} a_n = \infty$ and $a_{n+1} \leq a_n$ for $n \geq 1$.

C. $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \leq a_n$ for $n \geq 1$.

D. $\lim_{n\to\infty} a_n = \infty$ and $a_{n+1} \geq a_n > 0$ for $n \geq 1$.

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14. For what value of $k > 0$ will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{6}{k}\right)^n$ diverge?

A. 3

B. 4

C. 5

D. 7

Write your questions and thoughts here!

Find the values of x that make the series converge absolutely.

4. $\sum_{n=1}^{\infty} \frac{(-1)^n n(x+4)^n}{6^n}$

5. $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$

10.5 Absolute or Conditional Convergence

Calculus

Practice

1. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

A. I only

B. I and II only

C. I and III only

D. II and III only

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Determine whether the series converges absolutely, converges conditionally, or diverges.

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n^2+8)}{\pi^n}$

3. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

4. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(n+1)^2}$

5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{5}{2}}}$

6. For which values x is the series $\sum_{n=1}^{\infty} \frac{nx^n}{4^n(n^2+1)}$ conditionally convergent?

A. $x = 4$ B. $x = -4$ C. $x > 4$ D. $-4 < x < 4$

7. Which of the following statements is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/3}}$.

A. The series converges conditionally.

B. The series converges absolutely.

C. The series converges but neither conditionally nor absolutely.

D. The series diverges.

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8. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^{3/2}}$ is true?

A. The series converges conditionally.

B. The series converges absolutely.

C. The series converges but neither conditionally nor absolutely.

D. The series diverges.

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9. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$ is true?

I. Converges Absolutely   II. Diverges   III. Converges Conditionally

A. I only   B. II only   C. III only   D. I and III only

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10. For what values of x is the series $\sum_{n=1}^{\infty} \frac{n(x+5)^n}{7^n}$ absolutely convergent?

A. $x = -12$ B. $x = 2$ C. $x > 2$ D. $-12 < x < 2$

No test prep for this lesson.