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BC Calculus – 10.6 Notes – Ratio Test and Root Test

Recall:

$\frac{(n+1)!}{n!} =$

$\frac{3^{n+1}}{3^n} =$

Ratio Test for Convergence

If $\sum_{n=1}^{\infty} a_n$ has positive terms and…

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then the series
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$, then the series
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then

Let's look at two series we already know.

1. $\sum_{n=1}^{\infty} \frac{1}{n}$

2. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Using the Ratio Test to find convergence or divergence.

3. $\sum_{n=1}^{\infty} \frac{n^2 \cdot 3^{n+1}}{5^n}$

4. $\sum_{n=1}^{\infty} \frac{4^n}{n!}$

5. $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

The final test to determine convergence or divergence is the root test. The root test is especially well suited to solve series involving n^{th} powers.

Root Test

Let $\sum a_n$ be a series.

1) $\sum a_n$ converges absolutely if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$

2) $\sum a_n$ diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$

3) The Root Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$

Example: Using the Root Test

6) $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

7) $\sum_{n=1}^{\infty} \left(\frac{-3n}{2n+1}\right)^n$

8) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

9) $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

10) $\sum_{n=1}^{\infty} \left(\frac{3n+4}{2n}\right)^n$

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Write your questions and thoughts here!

5. $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

10.6 Ratio Test

Calculus

Practice

Determine whether the following series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{(n+1)3^n}{n!}$

2. $\sum_{n=1}^{\infty} \frac{n!}{5^n}$

3. What are values of $x > 0$ for which the series $\sum_{n=1}^{\infty} \frac{n6^n}{x^n}$ converges?

4. What are all positive values of p for which the series $\sum_{n=1}^{\infty} \frac{n^p}{7^n}$ will converge?

A. $p > 0$B. $0 < p < 7$

C. $p > 1$D. There are no positive values where the series will converge.

5. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{7^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{20}}$ III. $\sum_{n=1}^{\infty} \frac{\pi^{-2n}}{n}$

A. I only B. I and II only C. III only D. I and III only E. I, II, and III

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6. If the Ratio Test is applied to the series $\sum_{n=1}^{\infty} \frac{n\pi^n}{15^n}$, which of the following inequalities results, implying that the series converges?

A. $\lim_{n\to\infty} \frac{n\pi^n}{15^n} < 1$ B. $\lim_{n\to\infty} \frac{15^n}{n\pi^n} < 1$ C. $\lim_{n\to\infty} \frac{(n+1)\pi^{n+1}}{15^{n+1}} < 1$ D. $\lim_{n\to\infty} \frac{(n+1)\pi}{15n} < 1$

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7. If $a_n > 0$ for all n and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 6$, which of the following series converges?

A. $\sum_{n=1}^{\infty} a_n$ B. $\sum_{n=1}^{\infty} \frac{a_n}{n^7}$ C. $\sum_{n=1}^{\infty} \frac{a_n}{7^n}$ D. $\sum_{n=1}^{\infty} \frac{(a_n)^2}{7^n}$

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8. Consider the series $\sum_{n=1}^{\infty} \frac{n!}{3^n}$. If the Ratio Test is applied to the series, which of the following inequalities results, implying the series diverges?

A. $\lim_{n \to \infty} \frac{n!}{3^n} < 1$ B. $\lim_{n \to \infty} \frac{n!}{3^n} > 1$ C. $\lim_{n \to \infty} \frac{n+1}{3} < 1$ D. $\lim_{n \to \infty} \frac{n+1}{3} > 1$

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9. For which of the series is the Ratio Test inconclusive?

I. $\sum_{n=1}^{\infty} \frac{1}{3n}$ II. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$ III. $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

A. I only B. II only C. I and III only D. I and II only E. I, II, and III

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10. Apply any appropriate test to determine which of the following series diverges.

I. $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ II. $\sum_{n=1}^{\infty} \frac{n!}{9^n}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{4n+1}$

A. I only B. II only C. III only D. I and II only E. I, II, and III

Match the test for convergence of an infinite series with the conditions of convergence.

Convergence Test	Condition of convergence
11. _____ nth-Term Test	A. $p > 1$
12. _____ Geometric Series	B. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$
13. _____ p-series	C. $0 < a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ converges
14. _____ Alternating Series Test	D. $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges
15. _____ Integral Test	E. $\lvert r\rvert < 1$
16. _____ Ratio Test	F. Inconclusive for convergence
17. _____ Comparison Test	G. $\lvert a_{n+1}\rvert \le \lvert a_n\rvert$ and $\lim_{n\to\infty} a_n = 0$
18. _____ Limit Comparison Test	H. $\int_1^{\infty} f(x)\,dx$ converges.

10.6 Ratio Test

Test Prep

19. If the Ratio Test is applied to the series $\sum_{n=1}^{\infty} \frac{7^n}{(n+1)!}$, which of the following limits results, implying that the series converges?

A. $\lim_{n\to\infty} \frac{7^n}{(n+1)!}$ B. $\lim_{n\to\infty} \frac{7}{n+2}$ C. $\lim_{n\to\infty} \frac{(n+1)!}{7^n}$ D. $\lim_{n\to\infty} \frac{n+2}{7}$

20. Use the Ratio Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.