BC 2.1-2.5 Morning Review

Ex.1 $f(x) = \dfrac{2\sqrt{x}(x-3)^2}{3(\sqrt{x^7})}$

★ split into individual fractions, use power Rule (~~no~~ quotient Rule)

Avoid

$$f(x) = \frac{2x^{1/2}(x^2-6x+9)}{3x^{+7/2}}$$

$$f(x) = \frac{2x^{5/2} - 12x^{3/2} + 18x^{1/2}}{3x^{7/2}}$$

$$f(x) = \frac{2x^{5/2}}{3x^{7/2}} - \frac{12x^{3/2}}{3x^{7/2}} + \frac{18x^{1/2}}{3x^{7/2}}$$

$$f(x) = \frac{2}{3}x^{-2/2} - 4x^{-4/2} + 6x^{-6/2}$$

$$f(x) = \frac{2}{3}x^{-1} - 4x^{-2} + 6x^{-3}$$

$$f'(x) = -\frac{2}{3}x^{-2} + 8x^{-3} - 18x^{-4}$$

$$\boxed{f'(x) = \frac{-2}{3x^2} + \frac{8}{x^3} - \frac{18}{x^4}}$$

2) $f(x) = \begin{cases} bx^2 - 3, & x \le -1 \\ ax + b, & x > -1 \end{cases}$

Solve for a and b such that f(x) is differentiable at x = −1

* f(x) shares same y-value at x = −1 (~~set equations~~ (set expressions equal)

at x = −1

$ax + b = bx^2 - 3$

$a(-1) + b = b(-1)^2 - 3$

$-a + b = b - 3$

$-a = -3$

$\boxed{a = 3}$

f(x) must have same slope b/t piecewise functions at x = −1 (set derivatives equal)

$f'(x) = \begin{cases} 2bx, & x \le -1 \\ a, & x > -1 \end{cases}$

x = −1

$2bx = a$

$2b(-1) = a$

$-2b = a$

$a = -2b$

$3 = -2b$

$\boxed{\frac{-3}{2} = b}$

* General Definition of Derivative

$$* \; f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$* \; f(x+h) = -2(x+h)^2 - (x+h) + 1$$

find $f'(5)$ for

$$f(x) = -2x^2 - x + 1$$

$$f'(x) = \lim_{h\to 0}$$

$$f'(x) = \lim_{h\to 0} \frac{-2(x+h)^2 - (x+h) + 1 - (-2x^2 - x + 1)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{-2(x^2+2xh+h^2) - x - h + 1 + 2x^2 + x - 1}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{-2x^2} - 4xh - 2h^2 \cancel{-x} - h \cancel{+1} \cancel{+2x^2} \cancel{+x} \cancel{-1}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{-4xh - 2h^2 - h}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(-4x - 2h - 1)}{\cancel{h}}$$

$$f'(x) = -4x - 0 - 1$$

$$\boxed{f'(x) = -4x - 1}$$

$$f'(5) = -4(5) - 1 = \boxed{-21}$$

* Alternative def of derivative

$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(5) = \lim_{x\to 5} \frac{f(x) - f(5)}{x - 5}$$

$$f(5) = -2(5)^2 - 5 + 1$$

$$f(5) = -54$$

$$\lim_{x\to 5} \frac{-2x^2 - x + 1 - (-54)}{x - 5}$$

$$f'(5) = \lim_{x\to 5} \frac{-2x^2 - x + 55}{x - 5}$$

$$f'(5) = \lim_{x\to 5} \frac{-(2x^2 + x - 55)}{x - 5}$$

$$\frac{-(x + \frac{11}{2})(x - \frac{10}{2})}{x - 5}$$

$$\frac{-(2x+11)\cancel{(x-5)}}{\cancel{(x-5)}}$$

$$f'(5) = \lim_{x\to 5} -(2x + 11)$$

$$f'(5) = -(2(5) + 11) = \boxed{-21}$$