4.4 Exercise Problems Indeterminate Form and L'Hopital's Rule

p. 299-301 # 27-57 odd

27) $\lim_{x\to 2} \frac{x^2+x-6}{x^2-3x+2} \to \frac{0}{0} \to \lim_{x\to 2} \frac{2x+1}{2x-3} \to \frac{5}{+1} = \boxed{5}$

29) $\lim_{x\to 1} \frac{\ln x}{x^2-1} \to \frac{0}{0} \to \lim_{x\to 1} \frac{\frac{1}{x}}{2x} \to \frac{\frac{1}{1}}{2(1)} \to \boxed{\frac{1}{2}}$

31) $\lim_{x\to 0} \frac{e^x-e^{-x}}{\sin x} \to \frac{0}{0} \to \lim_{x\to 0} \frac{e^x-e^{-x}(-1)}{\cos x} \to \frac{e^0+e^0}{\cos(0)} = \frac{2}{1} = \boxed{2}$

33) $\lim_{x\to 1} \frac{\sin(\pi x)}{x-1} \to \frac{0}{0} \to \lim_{x\to 1} \frac{\cos(\pi x)\cdot \pi}{1} \to \frac{\pi\cos(\pi)}{1} \to \boxed{-\pi}$

37) $\lim_{x\to\infty} \frac{\ln x}{e^x} \to \frac{\infty}{\infty} \to \frac{\frac{1}{x}}{e^x} \to \lim_{x\to\infty} \frac{1}{xe^x} \to \boxed{0}$

* Also comparative Growth Rate $L < R < P < E$

35) $\lim_{x\to\infty} \frac{x^2}{e^x} \to \frac{\infty}{\infty} \to \lim_{x\to\infty} \frac{2x}{e^x} \to \boxed{0}$

39) $\lim_{x\to 0} \frac{e^x-1-\sin x}{1-\cos x} \to \frac{0}{0} \to \lim_{x\to 0} \frac{e^x-0-\cos x}{0-(-\sin x)} \to \frac{0}{0}$

$\lim_{x\to 0} \frac{e^x-(-\sin x)}{\cos x} \to \frac{e^0+\sin 0}{\cos(0)} \to \frac{1}{1} = \boxed{1}$

41) $\lim_{x\to 0} \frac{\sin x - x}{x^3} \to \frac{0}{0} \to \lim_{x\to 0} \frac{\cos x - 1}{3x^2} \to \frac{1-1}{0} \to \frac{0}{0}$

$\lim_{x\to 0} \frac{-\sin x - 0}{6x} \to \frac{0}{0} \to \lim_{x\to 0} \frac{-\cos x}{6} \to \boxed{\frac{-1}{6}}$

4.4

43) $\lim_{x\to 0^+} x^2 \ln x$

$$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} \to \frac{\infty}{\infty} \to \lim_{x\to 0^+} \frac{\ln x}{x^{-2}} \to \lim_{x\to 0^+} \frac{\frac{1}{x}}{-2x^{-3}}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} \to \lim_{x\to 0^+} \frac{1}{x}\cdot\frac{x^3}{-2} \to \lim_{x\to 0^+} \frac{x^2}{-2} \to \frac{0}{-2} = \boxed{0}$$

45) $\lim_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right) \to \lim_{x\to\infty} \frac{e^{1/x}-1}{1/x} \to \frac{0}{0} \to \lim_{x\to\infty} \frac{e^{x^{-1}}-1}{x^{-1}}$

$$\lim_{x\to\infty} \frac{e^{x^{-1}}\cdot -1x^{-2}}{-1x^{-2}} \to \lim_{x\to\infty} e^{1/x} \to \boxed{0}$$

47) $\lim_{x\to \pi/2} \sec x - \tan x \to \infty - \infty$

$$\lim_{x\to \pi/2} \frac{1}{\cos x} - \frac{\sin x}{\cos x} \to \lim_{x\to \pi/2} \frac{1-\sin x}{\cos x} \to \frac{0}{0}$$

$$\lim_{x\to \pi/2} \frac{-\cos x}{-\sin x} \to \frac{0}{1} = \boxed{0}$$

49) $\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{x}{\ln x}\right) \to \lim_{x\to 1} \frac{1-x}{\ln x} \to \frac{0}{0}$

$$\lim_{x\to 1} \frac{-1}{1/x} \to \lim_{x\to 1} -x \to \boxed{-1}$$

51) $\lim_{x\to 0^+} (2x)^{3x}$ let $y = (2x)^{3x}$

$\ln y = \ln(2x)^{3x}$

$\ln y = 3x \ln(2x)$

$\ln y = \lim_{x\to 0^+} 3x \ln(2x)$

$= \lim_{x\to 0^+} \frac{\ln(2x)}{\frac{1}{3x}} \to \frac{0}{0}$

$\to \frac{\ln(2x)}{(3x)^{-1}}$

$\lim_{x\to 0^+} \frac{\frac{2}{2x}}{-1(3x)^{-2}(3)} \to \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-3}{9x^2}}$

$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{3x^2}} \to \lim_{x\to 0^+} \frac{1}{x} \cdot \frac{-3x^2}{1} \to -3x$

$\lim_{x\to 0^+} -3x = 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y = 1}$

53) $\lim_{x\to\infty} (x+1)^{e^{-x}}$

$y = (x+1)^{e^{-x}}$

$\ln y = \ln(x+1)^{e^{-x}}$

$\ln y = e^{-x} \cdot \ln(x+1)$

$= \frac{\ln(x+1)}{e^x}$

$\lim_{x\to\infty} \frac{\ln(x+1)}{e^x} \to \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\frac{1}{x+1}}{e^x} \to \lim_{x\to\infty} \frac{1}{(x+1)e^x} \to \frac{1}{\infty} \to 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$\boxed{y = 1}$

55) $\lim_{x\to 0^+}(\csc x)^{\sin x}$

let $y=(\csc x)^{\sin x}$

$\ln y = \ln \csc^{\sin x}$

$\ln y = (\sin x)\ln(\csc x)$

$\ln y = \dfrac{\ln(\csc x)}{\csc x}$

$\ln y = \lim_{x\to 0^+} \dfrac{\ln(\csc x)}{\csc x} \to \dfrac{\infty}{\infty} \to \dfrac{\frac{-\csc x\cot x}{\csc x}}{-\csc x\cot x} \to \lim_{x\to 0} \dfrac{1}{\csc x} \to \sin x \to \sin 0 = 0$

$\ln y = 0$

$e^{\ln y} = e^{0}$

$y = e^0 = \boxed{1}$

57) $\lim_{x\to \frac{\pi}{2}^-}(\sin x)^{\tan x}$

$y = (\sin x)^{\tan x}$

$\ln y = \ln(\sin x)^{\tan x}$

$\ln y = \tan x \cdot \ln(\sin x)$

$\ln y = \dfrac{\ln(\sin x)}{\cot x}$

$\lim_{x\to \frac{\pi}{2}^-} \dfrac{\frac{\cos x}{\sin x}}{-\csc^2 x} \to \dfrac{\cos x}{\sin x}\cdot -\sin^2 x$

$\lim_{x\to \frac{\pi}{2}^-} \cos x \cdot -\sin x \to 0(-1) = 0$

$\ln y = 0$

$\boxed{y = 1}$