(8)

BC Calculus – 9.2 Notes – 2nd Derivative of Parametric Equations

Second Derivative of a Parametric Equation

The second derivative of a parametric given by $x = f(t)$ and $y = g(t)$ is

Given the following parametric equations, find $\frac{d^2y}{dx^2}$ in terms of t.

1. $x(t) = \sqrt{t}$ and $y(t) = \frac{1}{2}(t^2 - 2)$ for $t \geq 0$.

2. $x = 3\cos t$ and $y = 4\sin t$.

3. At $t = 1$, find the concavity of the graph defined parametrically by $x = t^3 + 1$ and $y = t^4 + t$.

9.2 practice problems

Given the following parametric equations, find $\frac{d^2y}{dx^2}$ in terms of t.

1. $x(t) = e^{-2t}$ and $y(t) = e^{2t}$.

2. $x(t) = t^3$ and $y(t) = t^4 + 1$ for $t > 0$.

3. $x(t) = at^3$ and $y(t) = bt$, where a and b are positive constants.

4. $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = \sin(t^2)$.

5. $x = e^t$ and $y = te^{-t}$.

6. $x = t^2 + 1$ and $y = 2t^3$.

7. Given a curve defined by the parametric equations $x(t) = 2 - t^2$ and $y(t) = t^2 + t^3$. Determine the open t-intervals on which the curve is concave up or down.

8. If $x(\theta) = 2 + \sec\theta$ and $y(\theta) = 1 + 2\tan\theta$, Find the slope and the concavity at $\theta = \frac{\pi}{6}$.

9. If $x = \cos\theta$ and $y = 3\sin\theta$, find the slope and concavity at $\theta = 0$.

10. If $x(t) = t - \ln t$ and $y(t) = t + \ln t$, determine values of t where the graph is concave up.

9.2 Second Derivatives of Parametric Equations

Test Prep

11. If $x = 3t^2 - 1$ and $y = \ln t$, what is $\frac{d^2y}{dx^2}$ in terms of t?

A. $\frac{1}{6}t^2$ B. $-\frac{1}{3}t^{-3}$ C. $-\frac{1}{18}t^{-4}$ D. $-\frac{1}{2}t^{-4}$ E. $6t^4$

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12. If $x = \theta - \cos\theta$ and $y = 1 - \sin\theta$, find the slope and concavity at $\theta = \pi$.

A. Slope: -1, Concave down B. Slope: π, Concave up C. Slope: 1, Concave down

D. Slope: 1, Concave up E. Slope: $\frac{1}{\pi}$, Concave up