(16)

BC Calculus – 9.3 Notes – Finding Arc Lengths (Parametric Equations)

Recall: Arc Length

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Arc Length in Parametric Form

For each set of parametric equations, find the length of the curve on the given interval.

1. $x(t) = \cos t$ and $y(t) = \sin t$ on the interval $0 \le t \le 2\pi$.

2. $x = 1 - 4t$ and $y = 7t$ on the interval $0 \le t \le 2$.

9.3 Arc Length (Parametric Form)

Calculus

Practice

What is the length of the curve defined by the parametric equations? Solve without the use of a calculator.

1. $x(t) = 6t + 10$ and $y(t) = 14 - 4t$ for the interval $-1 \leq t \leq 3$?

2. $x = \frac{a}{2}t^2$ and $y = \frac{b}{2}t^2$, where a and b are constants. What is the length of the curve from $t = 0$ to $t = 1$?

3. $x(t) = 2t^2$ and $y(t) = \frac{2}{3}t^3$ for the interval $1 \leq t \leq 4$?

4. $x(\theta) = 5\cos\theta$ and $y(\theta) = 5\sin\theta$ for the interval $0 \leq \theta \leq 2\pi$.

5. $x(t) = 7t - 2$ and $y(t) = 4 - 8t$ for the interval $1 \leq t \leq 5$.

6. If a curved is described by the parametric equations $x = t^2$ and $y = 2e^{2t}$, then which of the following gives the length of the path from $t = 0$ to $t = \ln 3$?

A. $\int_0^{\ln 3} \sqrt{4t^2 + 4e^{4t}}\, dt$

B. $\int_0^{\ln 3} \sqrt{t^4 + 4e^{4t}}\, dt$

C. $\int_0^{\ln 3} \sqrt{4t^2 + 16e^{4t}}\, dt$

D. $\int_0^{\ln 3} \sqrt{t^2 + 2e^{2t}}\, dt$

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7. Which of the following gives the length of the path described by the parametric equations $x = 2 + 4t$ and $y = 3 + t^2$ from $t = 0$ to $t = 1$?

A. $\int_0^1 \sqrt{4 + 2t}\, dt$

B. $\int_0^1 \sqrt{(2 + 4t)^2 + (3 + t^2)^2}\, dt$

C. $\int_0^1 \sqrt{16t^2 + t^4}\, dt$

D. $\int_0^1 \sqrt{16 + 4t^2}\, dt$

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8. Which of the following gives the length of the path described by the parametric equations $x = \cos t^3$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

A. $\int_0^{\pi} \sqrt{9t^4 \sin^2(t^3) + 25e^{10t}}\, dt$

B. $\int_0^{\pi} \sqrt{-3t^2 \sin(t^3) + 5e^{5t}}\, dt$

C. $\int_0^{\pi} \sqrt{9t^4 \sin^2(t^3) + 25e^{5t}}\, d$

D. $\int_0^{\pi} \sqrt{(\cos(t^3))^2 + (e^{5t})^2}\, dt$

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9. Which of the following gives the length of the path described by the parametric equations $x = \sin 3t$ and $y = \cos 2t$ from $t = 0$ to $t = \pi$?

A. $\int_0^{\pi} \sqrt{\sin^2 3t + \cos^2 2t}\, dt$

B. $\int_0^{\pi} \sqrt{\cos^2 3t + \sin^2 2t}\, dt$

C. $\int_0^{\pi} \sqrt{9\cos 3t + 4\sin 2t}\, dt$

D. $\int_0^{\pi} \sqrt{9\cos^2 3t + 4\sin^2 2t}\, dt$

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10. Which of the following gives the length of the path described by the parametric equations $x = \sqrt{t}$ and $y = 3t - 1$ from $0 \le t \le 1$?

A. $\int_0^1 \sqrt{\frac{t}{4} + 9}\, dt$

B. $\int_0^1 \sqrt{\frac{1}{4}t^{-1} + 9}\, dt$

C. $\int_0^1 \sqrt{\frac{1}{4}t + 3}\, dt$

D. $\int_0^1 \sqrt{\frac{1}{2}t^{-\frac{1}{2}} + 3}\, dt$

No test prep. Problems 6-10 are great examples of problems you may see on the AP Exam.