

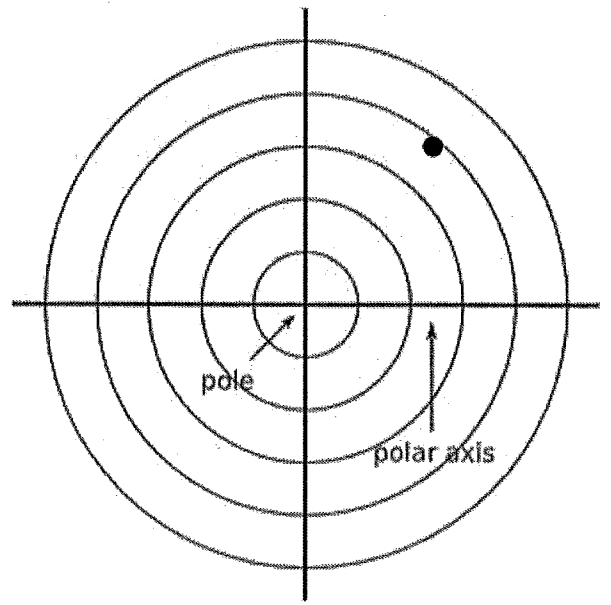
BC Calculus – 9.4-prep Notes – Graphing Polar Equations

A rectangular coordinate system is only one way to navigate through a Euclidean plane. Such coordinates, (x, y) , known as **rectangular coordinates**, are useful for expressing functions of y in terms of x . Curves that are not functions are often more easily expressed in an alternative coordinate system called **polar coordinates**.

In a polar coordinate system, we still have the traditional x - and y - axes. The intersection of these axes, the old origin, is called the **pole**. Similar to navigating on the Unit Circle, we can now get to any point in 2-D space by specifying an independent choice of an angle, θ , from the initial ray, **polar axis**, then walking out along that terminal ray a specified amount, r , in either direction.

Although the angle is the independent variable, we express

the point in the polar plane as (r, θ) . The point to the right would have coordinates of $\left(4, \frac{\pi}{4}\right)$.



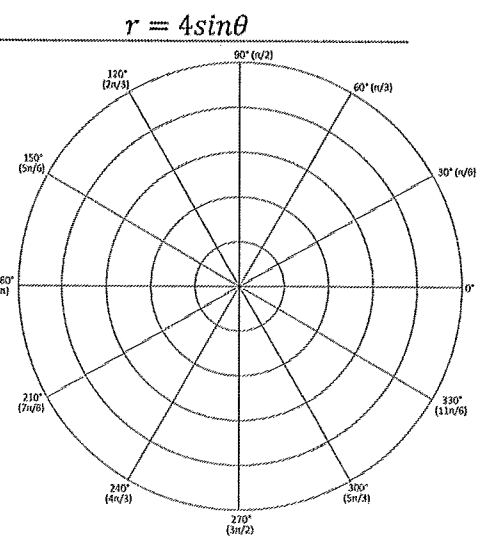
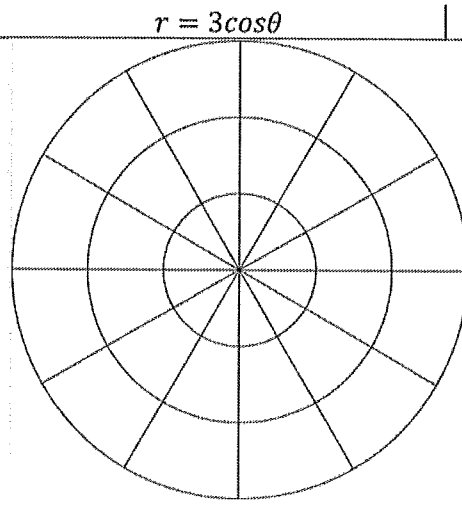
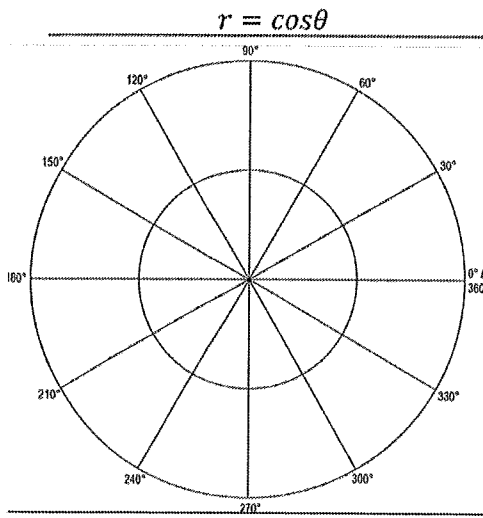
Example 1:

Find several other equivalent polar coordinates for the point shown above, then find the equivalent rectangular coordinate.

Why use polar coordinates? Graphs that aren't functions in rectangular form $f(x)$ can still be functions in polar form $r(\theta)$. Some of these curves can be quite elaborate and are more easily expressed as polar, rather than rectangular equations, as the following calculator exploration will demonstrate.

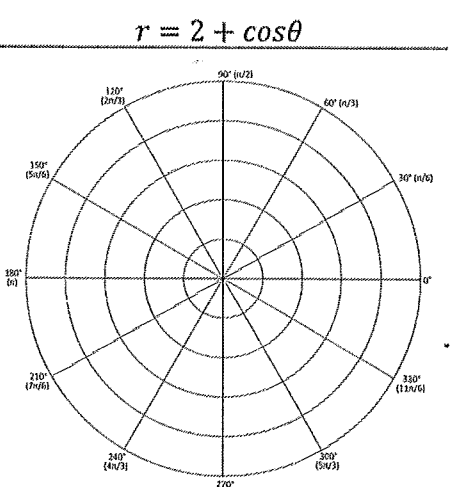
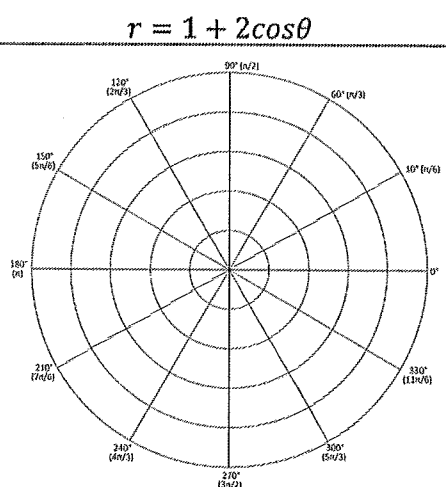
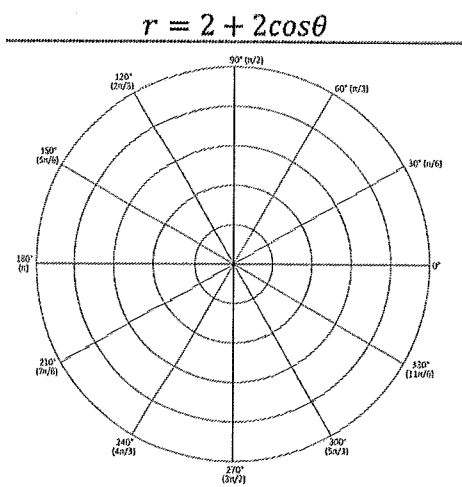
Example 2:

Put your graphing calculator in POLAR mode and RADIAN mode. Graph the following equations on your calculator, sketch the graphs on this sheet, and answer the questions.



What do you notice about the above graphs?

Example 3a:

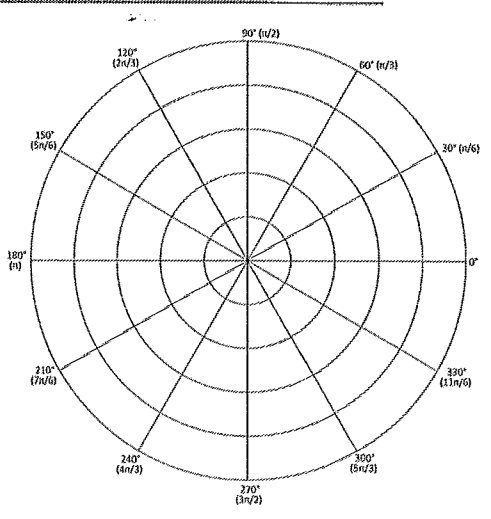
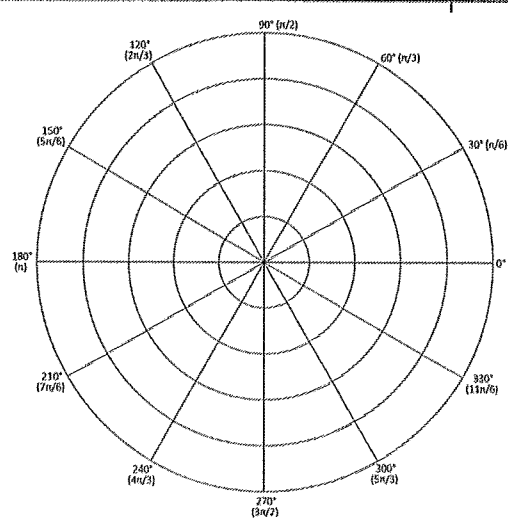
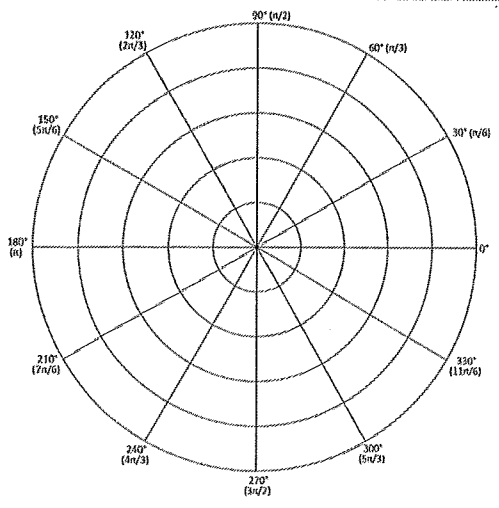


Example 3b:

$r = 2 + 2\sin\theta$

$r = 1 + 2\sin\theta$

$r = 2 + \sin\theta$



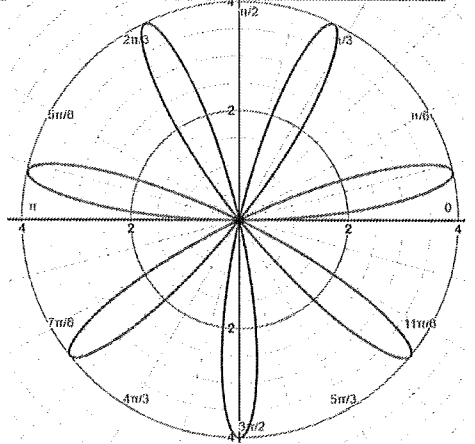
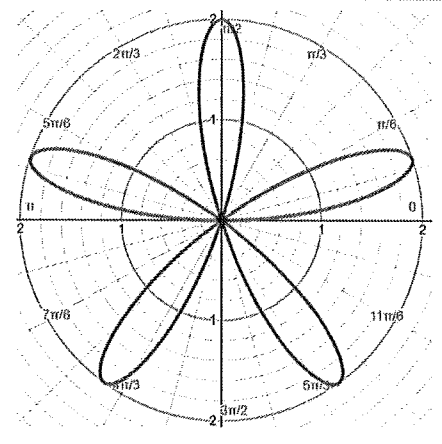
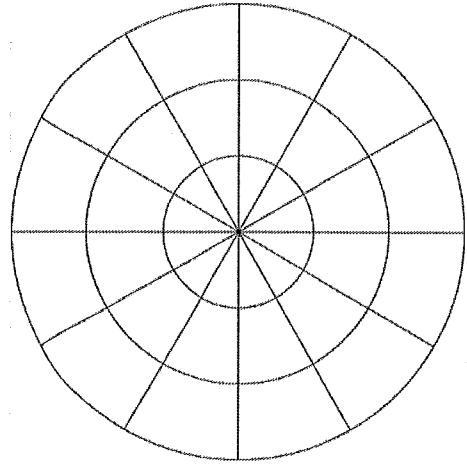
Which graphs go through the pole?
 Which ones do not go through the pole?
 Which ones have an inner loop?

Example 4:

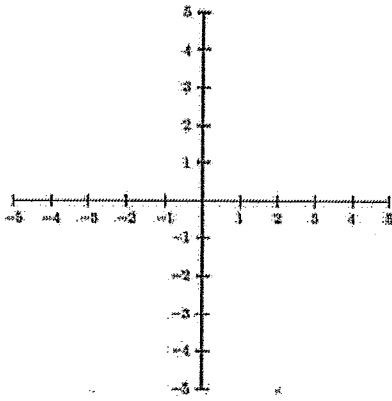
$r = 3\cos 3\theta$

$r = 2\sin 5\theta$

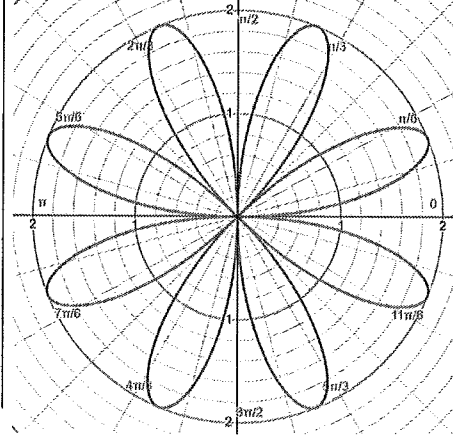
$r = 4\sin 7\theta$



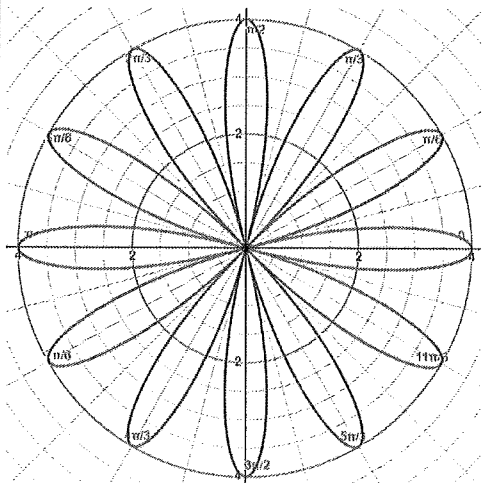
$$r = 3\cos 2\theta$$



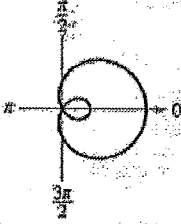
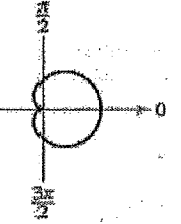
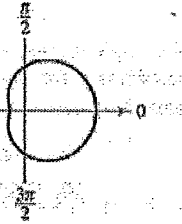
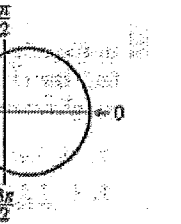
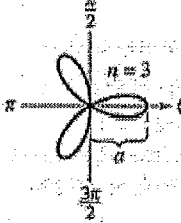
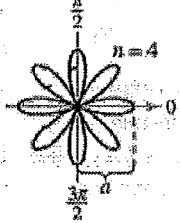
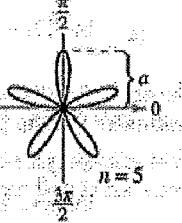
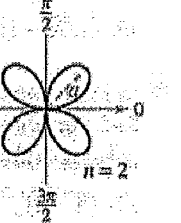
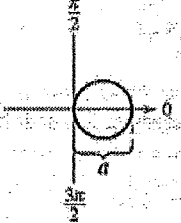
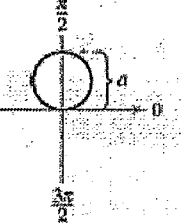
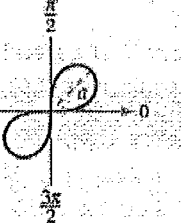
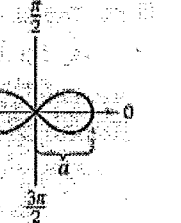
$$r = 2\sin 4\theta$$



$$r = 4\cos 6\theta$$



What do you notice about the above graphs?

<p>Limaçons</p> <p> $r = a \pm b \cos \theta$ $r = a \pm b \sin \theta$ $(a > 0, b > 0)$ </p>	 <p>$\frac{a}{b} < 1$ Limaçon with inner loop</p>	 <p>$\frac{a}{b} = 1$ Cardioid (heart-shaped)</p>	 <p>$1 < \frac{a}{b} < 2$ Dimpled limaçon</p>	 <p>$\frac{a}{b} \geq 2$ Convex limaçon</p>
<p>Rose Curves</p> <p>n petals if n is odd 2n petals if n is even</p>	 <p>$r = a \cos n\theta$ Rose curve</p>	 <p>$r = a \cos n\theta$ Rose curve</p>	 <p>$r = a \sin n\theta$ Rose curve</p>	 <p>$r = a \sin n\theta$ Rose curve</p>
<p>Circles and Lemniscates</p>	 <p>$r = a \cos \theta$ Circle</p>	 <p>$r = a \sin \theta$ Circle</p>	 <p>$r^2 = a^2 \sin 2\theta$ Lemniscate</p>	 <p>$r^2 = a^2 \cos 2\theta$ Lemniscate</p>