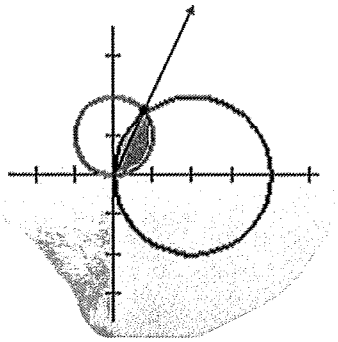
## BC Calculus – 9.4c Notes – Area Bounded by two Polar Curves

Recall area bounded by a polar curve: $A =$

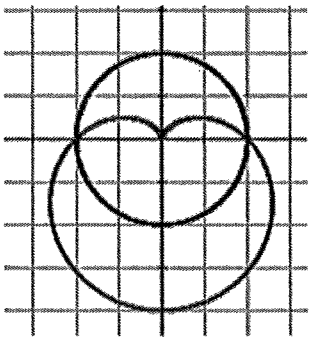
Things to watch for when using more than one polar curve for area.

- Points of intersection
- Symmetry

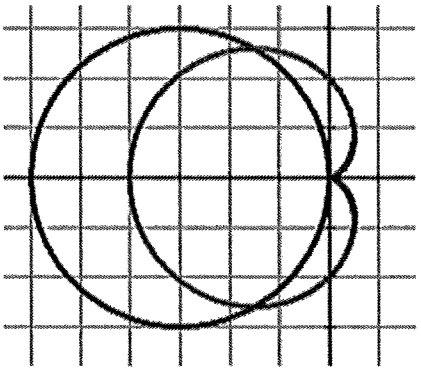
1. Find the area of the region common to the polar curve $r = 4\cos\theta$ and $r = 2\sin\theta$.

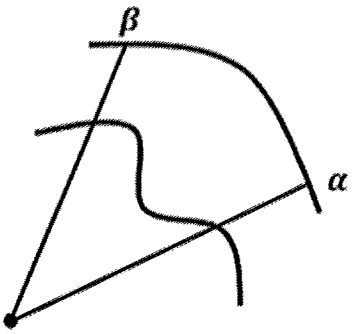


2. Find the area of the common region to the polar graphs of $r = 2$ and $r = 2 - 2\sin\theta$.



3. Find the area of the region common to the two polar curves $r = -6\cos\theta$ and $r = 2 - 2\cos\theta$.

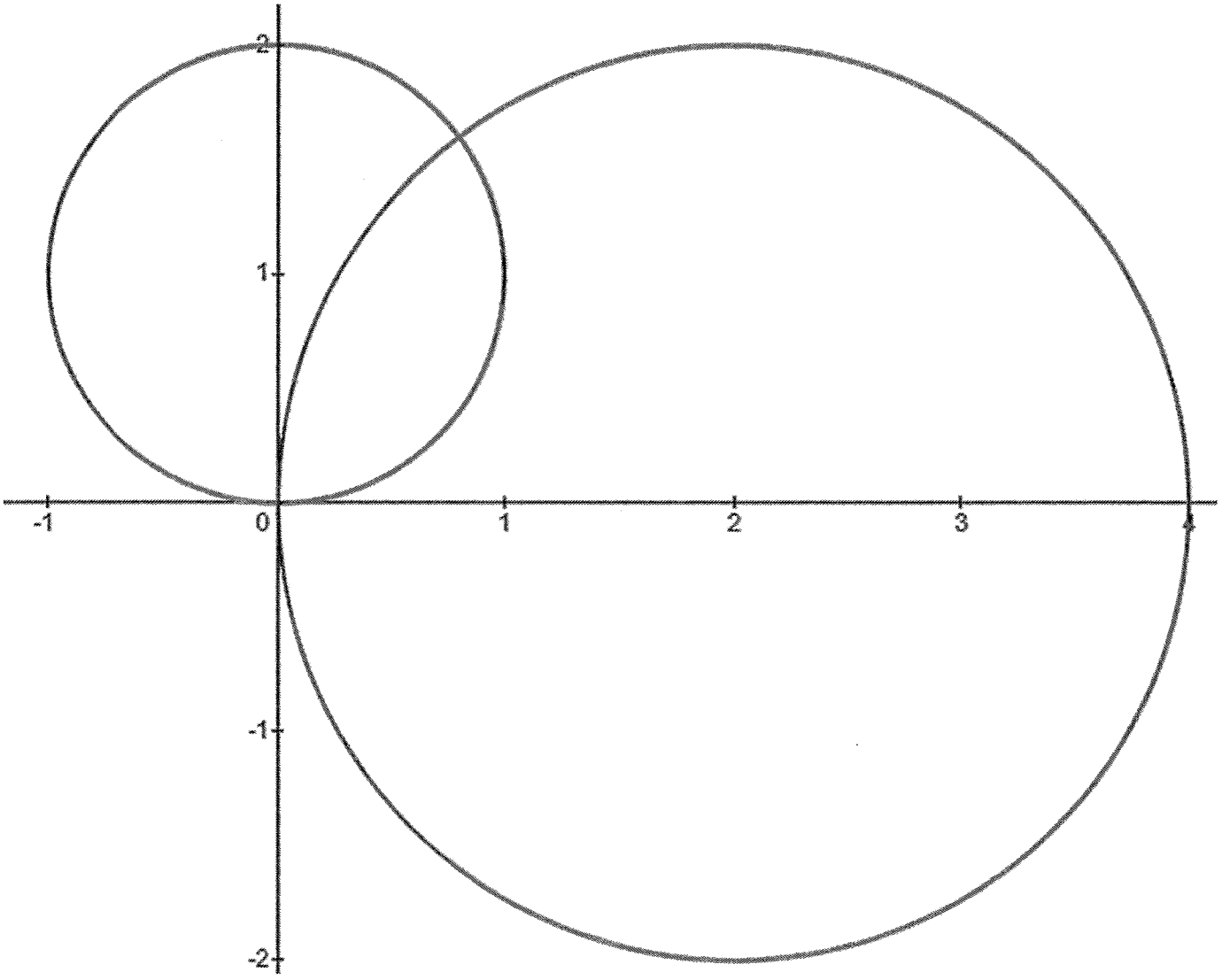




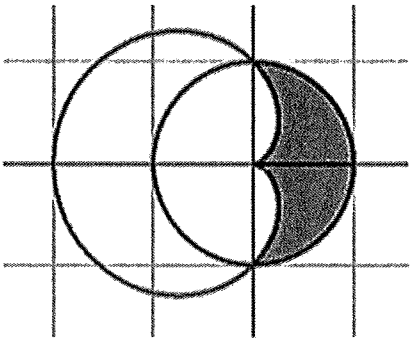

**Area Bounded by Two Polar Curves**

$A =$

$A =$


2
1
-1
0
1
2
3
4
-1
-2

4. Find the area of the region bounded by the two polar curves $r = 1$ and $r = 1 - \cos\theta$ as shown in the graph below.

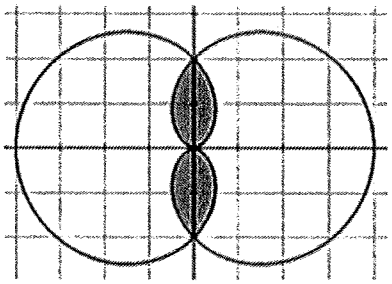


**Practice Problems:**

1. Find the area of the common interior of the polar graphs $r = 4\sin 2\theta$ and $r = 2$.

2. Find the area of the common interior of the polar graphs $r = 2\cos\theta$ and $r = 2\sin\theta$.

3. The polar curves $r = 2 - 2\cos\theta$ and $r = 2 + 2\cos\theta$ are shown below.



Which of the following gives the total area of the shaded regions?

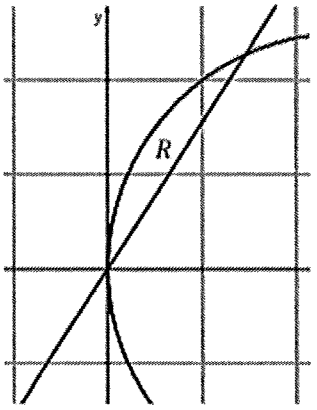
A. $\int_0^{\pi}(2 + 2\cos\theta)^2\ d\theta$

B. $\int_{\pi/2}^{\pi}(2 + 2\cos\theta)^2\ d\theta$

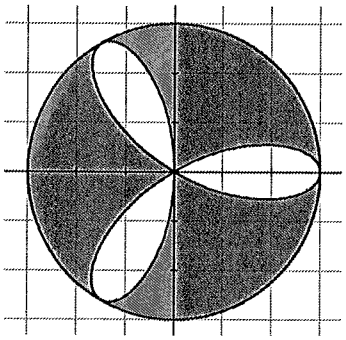
C. $8\int_0^{\frac{\pi}{2}}(1 - \cos\theta)^2\ d\theta$

D. $\int_0^{\frac{\pi}{2}}((2 - 2\cos\theta)^2 + (2 + 2\cos\theta)^2)\ d\theta$

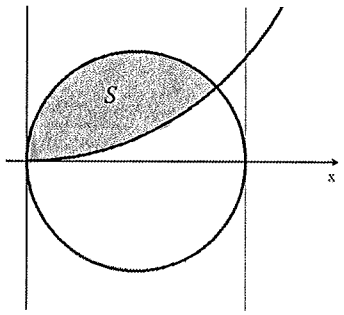
4. Let $R$ be the region in the first quadrant that is bounded above by the polar curve $r = 5\cos\theta$ and below by the line $\theta = 1$, as shown in the figure below. What is the area of $R$?



5. The figure below shows the graphs of the polar curves $r = 3\cos 3\theta$ and $r = 3$. What is the sum of the areas of the shaded regions?



6. Let $S$ be the region in the 1st Quadrant bounded above by the graph of the polar curve $r = \cos\theta$ and bounded below by the graph of the polar curve $r = \frac{5}{2}\theta$, as shown in the figure above. The two curves intersect when $\theta = 0.373$. What is the area of $S$?




7. Find the area inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = 1$.

8. Write an integral expression that represents the area of the region outside the polar curve $r = 3 + 2\sin\theta$ and inside the polar curve $r = 2$.

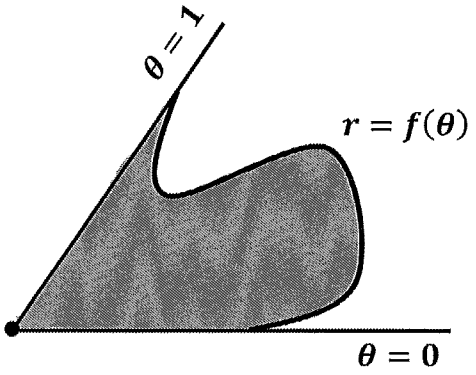
9. What is the total area outside the polar curve $r = 5\cos 2\theta$ and inside the polar curve $r = 5$?

10. Find the area of the common interior of the polar curves $r = 4\sin\theta$ and $r = 2$.

## 9.4 Area Bounded by Two Polar Curves

**Test Prep**

11.




| $\theta$ | 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |
|---|---|---|---|---|---|
| $r$ | 1 | 3 | 5 | 4 | 2 |

**No calculator!** Let $R$ be the region bounded by the graph of the polar curve $r = f(\theta)$ and the lines $\theta = 0$ and $\theta = 1$, as shaded in the figure above. The table above gives values of the polar function $r = f(\theta)$ at selected values of $\theta$. What is the approximation for the area of region $R$ using a right Riemann sum with the four subintervals indicated by the data in the table?