BC Calculus – 9.6 Motion Using Parametrics and Vectors Notes

Position: $r(t) = \langle x(t), y(t) \rangle$

Velocity: $v(t) = r'(t) = \langle x'(t), y'(t) \rangle$

Acceleration: $a(t) = r''(t) = \langle x''(t), y''(t) \rangle$

Speed: $\|v(t)\| = \|r'(t)\| =$

1. Find the velocity vector, speed, and acceleration vector for the particle that moves in the xy-plane described by $r(t) = \langle 5 \sin \frac{t}{5}, 5 \cos \frac{t}{5} \rangle$

Quick review: When does a particle's speed increase or decrease?

Speeding up

Velocity & Acceleration have

Slowing down

Velocity & Acceleration have

2. If $r(t) = \langle 2t^3 + t, t^2 \rangle$, find velocity and acceleration at time t.

3. Find the speed at time $t = 2$ if $r(t) = \langle 3t, e^{-t^2} \rangle$

Total Distance Traveled by a Partice on $[a, b]$.

$$\int_a^b \|v(t)\| \, dt =$$

4. Given the velocity vector of the particle $v(t) = \langle 2t + 1,\ 5\rangle$ and the position of the particle at time $t = 0$ is $(1, 2)$, find the position when $t = 3$. What is the total distance traveled on the interval $0 \le t \le 3$?

5. A particle moving along a curve so that its velocity for time $t \ge 0$ is given by $v(t) = \langle 2e^{-\frac{t}{4}}, \frac{t-4}{t+5}\rangle$.
   a. For what values of t is the particle moving to the right?

   b. For what values of t is the particle moving up?

Practice Problems:

For each problem, a particle moves in the xy-plane where the coordinates are defined at any time t by the position function given in parametric or vector form.	
1. $x(t) = 4t^2$ and $y(t) = 2t - 1$. Find the velocity vector at time $t = 1$.	2. $x(t) = e^{-t}$ and $y(t) = e^{t}$. Find the acceleration vector at time $t = 1$.

3. $(x(t), y(t)) = (6 - 2t, t^2 + 3)$. In which direction is the particle moving as it passes through the point $(4, 4)$?

4. A position vector is $r(t) = \langle \frac{2}{t},\ e^{4t} \rangle$ for time $t > 0$. What is the velocity vector at time $t = 1$?

5. $r(t) = \langle \ln(t^2 + 1), 3t^2 \rangle$ for $t > 0$. Find the velocity vector at time $t = 2$.

6. $x(t) = 2 \sin \frac{t}{2}$ and $y(t) = 2 \cos \frac{t}{2}$ for time $t > 0$. Find the speed of the particle.

7. **Calculator active.** $x(t) = t^2 + 1$ and $y(t) = \frac{4}{3}t^3$ for time $t \geq 0$. Find the total distance traveled from $t = 0$ to $t = 3$.

8. $p(t) = \langle \cos 2t, 2 \sin t \rangle$. Find the velocity vector $v(t)$.

9. **Calculator active.** The velocity vector of a particle moving in the xy-plane has components given by $\frac{dx}{dt} = \cos t^2$ and $\frac{dy}{dt} = e^{t-2}$. At time $t = 3$, the position of the particle is $(1, 2)$. What is the y-coordinate of the position vector at time $t = 2$?

10. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^3, 4t \rangle$. What is the acceleration vector when $t = 2$?

11. The acceleration vector of a particle moving in the xy-plane is given by $a(t) = \langle 2, 3 \rangle$. When $t = 0$ the velocity vector is $\langle 3, 1 \rangle$ and the position vector is $\langle 1, 5 \rangle$. Find the position when time $t = 2$.

12. A particle moves on the curve $y = 2x$ so that the x-component has velocity $x'(t) = 3t^2 + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(2, 4)$. At what point is the particle when $t = 1$? [This one is tricky!]

For problems 13-15: At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by parametric equations $x(t) = \cos 2t$ and $y(t) = \sin 2t$.

13. Find the speed of the particle when $t = 1$.

14. Find the acceleration vector at time $t = \frac{\pi}{4}$.

15. Find the distance traveled from $t = 0$ to $t = 3$.

9.6 Motion using Parametric and Vector-Valued Functions

Test Prep

16. **Calculator active.** A remote-controlled car moves along a flat surface over the time interval $0 \le t \le 30$ seconds. The position of the remote-controlled car at time t is given by the parametric equations $x(t) = 2t + \sin t$ and $y(t) = 2\cos(t - \sin t)$, where $x(t)$ and $y(t)$ are measured in feet. The derivatives of these functions are given by $x'(t) = 2 + \cos t$ and $y'(t) = -2\sin(t - \sin t)(1 - \cos t)$.

    a. Write the equation for the line tangent to the path of the remote-controlled car at time $t = 3$ seconds.

    b. Find the speed of the remote-controlled car at time $t = 15$ seconds.

    c. Find the acceleration vector of the remote-controlled car at the time when the car is at the point with x-coordinate 40.