Key

BC Calculus - Unit 1 Test Review (Limits)

Evaluate the Limit

1) $\lim\limits_{x\to 0^-} \frac{\sqrt{x+19}-\sqrt{19}}{x} \to \frac{0}{0}$

$\lim\limits_{x\to 0} \frac{\sqrt{x+19}-\sqrt{19}}{x} \cdot \frac{(\sqrt{x+19}+\sqrt{19})}{\sqrt{x+19}+\sqrt{19})}$

$\lim\limits_{x\to 0} \frac{\cancel{x}+\cancel{19}-\cancel{19}}{\cancel{x}(\sqrt{x+19}+\sqrt{19})} \to \lim\limits_{x\to 0} \frac{1}{\sqrt{x+19}+\sqrt{19}} \to \boxed{\frac{1}{2\sqrt{19}}}$

2) $\lim\limits_{x\to -3} \frac{x+3}{x^2+2x-3} \to \frac{0}{0}$

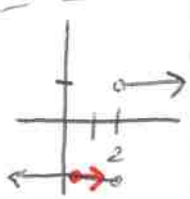
$\lim\limits_{x\to -3^+} \frac{\cancel{(x+3)}\ 1}{\cancel{(x+3)}(x-1)} \to \frac{1}{-3-1} \to -\frac{1}{4} \to \boxed{\frac{-1}{4}}$

3) $\lim\limits_{x\to 2^-} \frac{|x-2|}{x-2} \to \frac{|1.9-2|}{1.9-2} \to \frac{|-0.1|}{-0.1}$

*test $x=1.9$

$= \boxed{-1}$

$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2}, & x>2 \\ \frac{-(x-2)}{x-2}, & x<2 \end{cases} \to \begin{cases} 1, & x>2 \\ -1, & x<2 \end{cases}$



4) $\lim\limits_{x\to 0} \frac{\frac{1}{x+1}-1}{x} \to \frac{0}{0}$

$\lim\limits_{x\to 0} \frac{\frac{1}{x+1}-\frac{x+1}{x+1}}{x}$

$\lim\limits_{x\to 0} \frac{\frac{1-x-1}{x+1}}{x}$

$\lim\limits_{x\to 0} \frac{\frac{-x}{x+1}}{x}$

$\lim\limits_{x\to 0} \frac{-\cancel{x}}{x+1} \cdot \frac{1}{\cancel{x}}$

$\lim\limits_{x\to 0} \frac{-1}{x+1} \to -\frac{1}{1} = \boxed{-1}$

5) $\lim\limits_{x\to\infty} \frac{4x^5-2x^2+3}{3x^2+2x^5-x^4} = \frac{4}{2} = \boxed{2}$

*N = D

6) $\lim\limits_{x\to\infty} x^5 3^{-x}$

$\lim\limits_{x\to\infty} \frac{x^5}{3^x} = \boxed{0}$

*comparative growth Rate
L < R < P < E

$\lim\limits_{x\to\infty} \frac{\text{slower}}{\text{faster}} = 0$

7) $\lim\limits_{x\to -3^-} \frac{x^2+3}{x+3} \to \frac{12}{0}$ V.A. at $x=-3$

*test $x=-3.1$

$\frac{(-3.1)^2+3}{-3.1+3} \to \frac{+}{-} \to \boxed{-\infty}$

8) $\lim\limits_{x\to 1^+} \frac{x^2+2x+1}{x-1} \to \frac{4}{0} \to$ V.A. at $x=1$

*test $x=1.1$

$\frac{(1.1)^2+2(1.1)+1}{1.1-1} \to \frac{+}{+} \to \boxed{+\infty}$

9) Let g and h be the functions defined by $g(x) = -\frac{1}{4}x^2 - \frac{1}{2}x - \frac{9}{4}$ and $h(x) = \sin\left(\frac{\pi}{2}x\right) - 1$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for all x, what is $\lim_{x\to -1} f(x)$?

*Squeeze theorem

$$\lim_{x\to -1} \frac{-1}{4}x^2 - \frac{1}{2}x - \frac{9}{4} \le \lim_{x\to -1} f(x) \le \lim_{x\to -1} \sin\left(\frac{\pi}{2}x\right) - 1$$

$$\frac{-1}{4} + \frac{1}{2} - \frac{9}{4} \le \lim_{x\to -1} f(x) \le -1 - 1$$

$$-2 \le \lim_{x\to -1} f(x) \le -2$$

By squeeze theorem, $\boxed{\lim_{x\to -1} f(x) = -2}$

10) Let f be the function defined by $f(x) = \begin{cases} \frac{x^2+8x+12}{x+6}, & x \ne -6 \\ b, & x = -6 \end{cases}$. For what value of b is f continuous at $x = -6$?

(support answer with continuity conditions)

i) $f(-6) = b$

ii) $\lim_{x\to -6} \frac{x^2+8x+12}{x+6} \to \frac{0}{0} \to \lim_{x\to -6} \frac{\cancel{(x+6)}(x+2)}{\cancel{(x+6)}} \to -4$

iii) $f(-6) = \lim_{x\to -6} f(x)$

$\downarrow$

$\boxed{b = -4}$

Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined.*"

11. $\lim_{x\to 3} f(x) = \boxed{2}$

12. $\lim_{x\to 1} f(x) = \boxed{4}$

13. $f(3) = \boxed{\text{undefined}}$

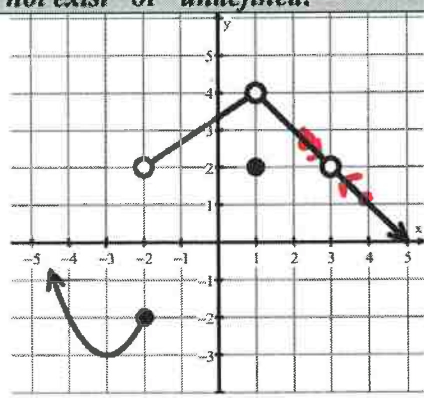
14. $f(-2) = \boxed{-2}$

15. $\lim_{x\to 2} f(x) = \boxed{3}$

16. $\lim_{x\to -2^+} f(x) = \boxed{2}$

17. $f(1) = \boxed{2}$

18. $\lim_{x\to -2^-} f(x) = \boxed{-2}$



19) If $f(x) = \begin{cases} \sin x, & x < -\pi \\ \tan x & -\pi < x < \frac{\pi}{4} \\ \cos x, & x \ge \frac{\pi}{4} \end{cases}$, find the following:

a. $\lim_{x\to -\pi^-} f(x) =$

b. $\lim_{x\to -\pi} f(x) =$

c. $\lim_{x\to \frac{\pi}{4}} f(x) =$

d. $f\left(\frac{\pi}{4}\right) =$

a) $\lim_{x\to -\pi^-} \sin x \to \sin(-\pi) = \boxed{0}$

b) $\lim_{x\to -\pi^+} \tan x \to \tan(-\pi) = 0$

* since $\lim_{x\to -\pi^-} f(x) = \lim_{x\to -\pi^+} f(x)$

then $\lim_{x\to -\pi} f(x) = 0$

c) $\lim_{x\to \pi/4^-} \tan x = \tan(\pi/4) = 1$

$\lim_{x\to \pi/4^+} \cos x = \cos(\pi/4) = \frac{\sqrt{2}}{2}$

Since $\lim_{x\to \pi/4^-} f(x) \ne \lim_{x\to \pi/4^+} f(x)$,

then $\lim_{x\to \pi/4} f(x) = \boxed{\text{dne}}$

d) $f(\pi/4) = \cos(\pi/4)$

$= \boxed{\frac{\sqrt{2}}{2}}$

20) If $f(x) = \frac{x+3}{x^2-2x-15}$, identify the type of each discontinuity and where it is located.

$f(x) = \frac{(\cancel{x+3})}{(x-5)(\cancel{x+3})}$

$x-5=0$ $\quad x+3=0$

$x=5$ $\quad x=-3$

VA → vertical asymptote at $x=5$ — Nonremovable discontinuity at $x=5$

hole → hole at point $(-3, \frac{-1}{8})$ — Removable Discontinuity at $x=-3$.

21) Identify all horizontal asymptotes of $f(x) = \frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x}$

$(x^6)^{1/2} = x^3$

$\lim_{x\to\infty} \frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x} = \frac{\sqrt{16}}{5} \to \frac{4}{5}$

$\lim_{x\to-\infty} \frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x} \to \frac{\sqrt{16}}{-5} \to \frac{4}{-5}$

$\boxed{\text{Horizontal Asymptotes occur at } y=\frac{4}{5} \text{ and } y=\frac{-4}{5}}$

22)

$$g(x) = \begin{cases} \frac{x+5}{2x-4}, x<2 \\ 12, x=2 \\ 2x-5,\ 2<x<5 \\ 20, x=5 \\ \frac{-x^2+4}{5-x}, x>5 \end{cases}$$

Find the following :

a) $\lim_{x\to-\infty} g(x) =$ $\lim_{x\to-\infty} \frac{x+5}{2x-4} \to \boxed{\frac{1}{2}}$	b) $\lim_{x\to 2^-} g(x) =$ test $x=1.9$ $\lim_{x\to 2^-} \frac{x+5}{2x-4} \to \frac{7}{0} \to \frac{+}{-} \to \boxed{-\infty}$	c) $\lim_{x\to 2^+} g(x) =$ $\lim_{x\to 2^+} 2x-5 \to 4-5 = \boxed{-1}$
d) $\lim_{x\to 2} g(x) = \boxed{dne}$ $\lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$	e) $\lim_{x\to 5^-} g(x) =$ $\lim_{x\to 5^-} 2x-5 = 10-5 = \boxed{5}$	f) $\lim_{x\to 5^+} g(x) =$ test $x=5.1$ $\lim_{x\to 5^+} \frac{-x^2+4}{5-x} \to \frac{-21}{0} \to \frac{-(5.1)^2+4}{5-5.1} \to \frac{-}{-} \to \boxed{+\infty}$
g) $\lim_{x\to 5} g(x) = \boxed{dne}$ $\lim_{x\to 5^-} g(x) \neq \lim_{x\to 5^+} g(x)$	h) $\lim_{x\to 3^+} g(x) =$ $\lim_{x\to 3^+} 2x-5 \to 6-5 = \boxed{1}$	i) $\lim_{x\to\infty} g(x) =$ *N > D $\lim_{x\to\infty} \frac{-x^2+4}{5-x}$ → $+\infty$ / → $-\infty$ test $x=100$ $\frac{-(100)^2+4}{5-100} \to \frac{-}{-} \to \boxed{+\infty}$

23) On the coordinate plane below, sketch a **function graph** with the following characteristics:

a. $\lim_{x \to -\infty} f(x) = 8$

b. $f(-6) = -3$

c. $\lim_{x \to -6} f(x) = 4$

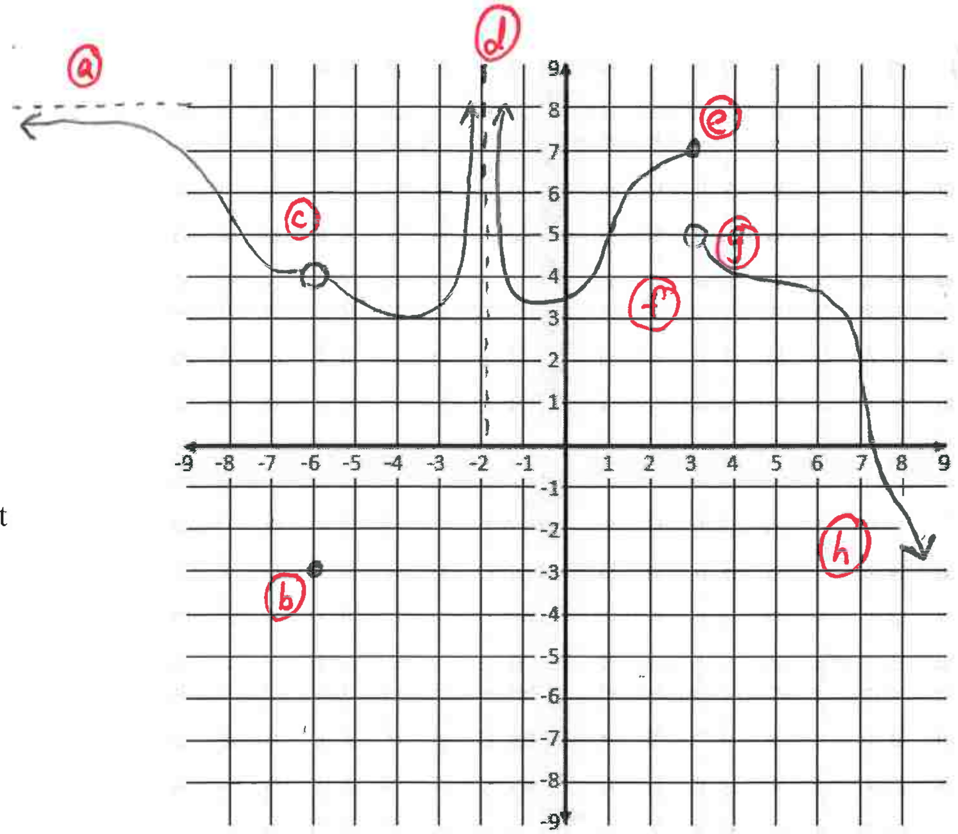
d. $\lim_{x \to -2} f(x) = \infty$

e. $f(3) = 7$

f. $\lim_{x \to 3} f(x)$ does not exist

g. $\lim_{x \to 3^+} f(x) = 5$

h. $\lim_{x \to \infty} f(x) = -\infty$



24) Find the value of k which makes the following piecewise function continuous for all values of x. (Use continuity conditions to justify)

$$f(x) = \begin{cases} 2x + k, & \text{if } x \le -2 \\ kx - 3, & \text{if } x > -2 \end{cases}$$

i) $f(-2) = 2(-2) + k \rightarrow -4 + k$

ii) $\lim_{x \to -2^-} 2x + k = -4 + k$ $\quad \lim_{x \to -2^+} kx - 3 = -2k - 3$

Since $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x) \rightarrow -4 + k = -2k - 3$

$-1 = -3k \rightarrow \boxed{k = \frac{1}{3}}$

25) See Next Page

23) On the coordinate plane below, sketch a **function graph** with the following characteristics:

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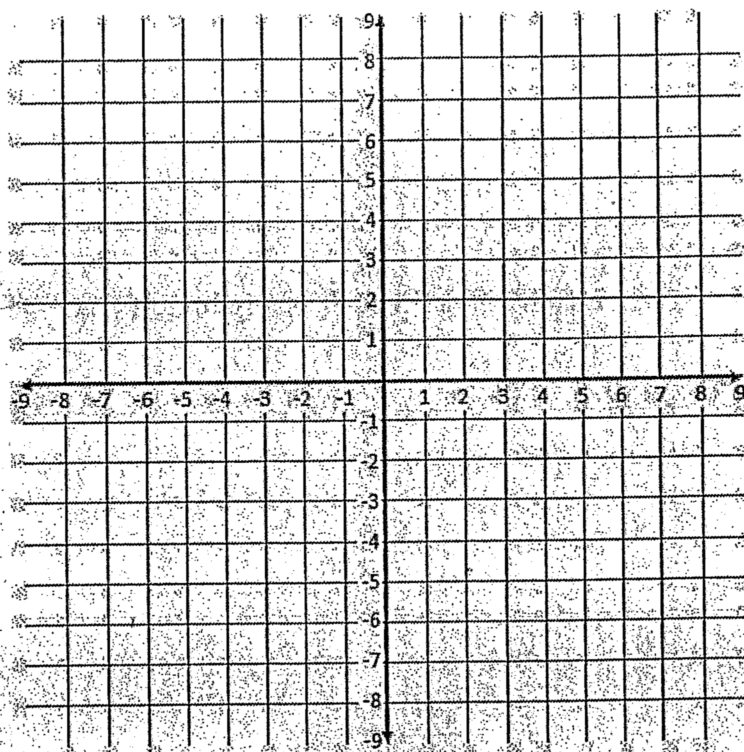

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$$f(x) = \begin{cases} 2x + k & \text{if } x \leq -2 \\ kx - 3 & \text{if } x > -2 \end{cases}$$

25) Verify IVT applies to $f(x) = \frac{x^2+x}{x-1}$ on $[5/2, 4]$ for $f(c)=6$

a)

b) Find c

a) $f(x)$ continuous on $[5/2, 4]$

VA at $x=1$

$f(5/2) = \frac{2.5^2+2.5}{2.5-1} = \frac{35}{6} \approx 5.8$

$f(4) = \frac{4^2+4}{4-1} = \frac{20}{3} \approx 6.7$

a) By IVT, since $f(5/2) = \frac{35}{6} < f(c) = 6 < \frac{20}{3} = f(4)$,

$f(c) = 6$ on interval $[5/2, 4]$

b) *set $f(x)=6$, solve for x

$\frac{x^2+x}{x-1} = \frac{6}{1}$

$x^2+x = 6(x-1)$

$x^2+x-6x+6=0$

$x^2-5x+6=0$

$(x-3)(x-2)=0$

$x=3$, ~~$x=2$~~

$\boxed{c=3}$ on $[5/2, 4]$