BC Calculus - Unit 3 - Composite, Implicit, & Inverse Functions - Test Review Worksheet

Show all appropriate work!

Find the derivative.

1. $h(x) = \cos^2(4x)$

2. $y = \ln\sqrt{x+3}$

3. $x^2 + 2y^5 = 10xy$

4. $y = \csc^{-1}(x^3)$

For each problem, let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x.

5. $f(6) = -1$, $f(4) = -2$, $f'(6) = 3$, and $f'(4) = 7$. What is the value of $g'(-1)$?

6. Let f be the function defined by $f(x) = x^3 + 3x + 1$. Let $g(x) = f^{-1}(x)$, where $g(-3) = -1$. What is the value of $g'(-3)$?

Find $\frac{d^2y}{dx^2}$ based on the given information.

7. $y = x^5 - e^{4x}$

8. $y = y^2 + x$

9. Find the equation of the tangent line.
$x^2 + 7y^2 = 8y^3$ at $(-6, 2)$

10. If $x = y^2 - \cos x$ find $\frac{d^2y}{dx^2}$ at $(0, -1)$.

10. Find the equation of any horizontal tangent lines for the graph of $(y^3 + 1)^2 = x^2 + 4x + 4$.

11. Slope of the tangent line of $g(x) = 4\sin^3 x$ at $x = \frac{\pi}{4}$.

12. Let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x. $f(6) = 8$, $f(8) = 2$, $f'(6) = -3$, and $f'(8) = 4$. What is the value of $g'(8)$?

Find $\frac{d^2y}{dx^2}$ based on the given information.

13. $y = e^{x^4}$

14. $5y^2 + 3 = x^2$

Evaluate the 2nd derivative at the given point.

15. If $f(x) = x^3 + \frac{5}{x}$, find $f''(-1)$.

16. If $x^2 + y^2 = 13$, find $\frac{d^2y}{dx^2}$ at $(2, 3)$.

The table below gives values of the differentiable functions g and h, as well as their derivatives, g' and h', at selected values of x.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
-1	0	4	3	6
0	9	2	0	-4
3	-1	-2	9	4
9	3	1	16	9

17. If $f(x) = \frac{g(x)}{\sqrt{h(x)}}$, find $f'(3)$.

18. Find $\frac{d}{dx}h^{-1}(9)$.

19. Find the equation of the tangent line to $g^{-1}(x)$ at $x = 3$.

Additional Practice Problems

Find $\frac{dy}{dx}$.

1. $y = \frac{e^{\tan 3x}}{3}$

2. $y = \ln(\sin 5x)$

3. $y = x\ln(4x)$

4. $e^{y^2} = x^5 + 10$

5. $y = \cos^{-1}(7x^3)$

6. $2x^3 - xy = \ln(y)$

Find the equation of the tangent line at the given point.

7. $4x^3 = -5xy + 4y$ at $(1, -4)$

8. $y = \arccos(5x)$ at $x = -\frac{\sqrt{3}}{10}$

9. $h(x) = (2x-1)^3(x+2)$ at $x = 1$.