Key

BC Calculus Unit 5 Curve Sketching Test Review Worksheet

1) If $y = -2x^2 + 4x + 3$ apply the Mean Value Theorem to find when the instantaneous rate of change will equal the average rate of change on the interval $[1, 3]$.

$y(x)$ continuous on $[1,3]$, differentiable on $(1,3)$

$y(1) = -2+4+3 = 5$

$y(3) = -18+12+3 = -3$

Avg. RoC $= \dfrac{-3-5}{3-1} = \dfrac{-8}{2} = -4$

$y'(x) = -4x + 4$

*set $y'(x) =$ Avg. RoC

$-4 = -4x + 4$

$-8 = -4x$

$x = 2$

$\boxed{c = 2}$

2) What is the absolute maximum value AND the absolute minimum value of the function $g(x) = x^3 - 12x$ on the closed interval $[0, 4]$.

y-value

(Apply Extreme Value Theorem steps and justification)

$g(x)$ continuous $[0,4]$

$g'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$0 = 3(x+2)(x-2)$

$x = \cancel{-2}, 2$

$g(0) = 0$

$g(2) = 8 - 24 = -16$

$\cancel{g(-2) = -8 + 24 = +16}$

outside interval

$g(4) = 4^3 - 48 = 16$

Abs max value is 16

Abs min value is -16

3) Find the intervals of concavity for the function $f(x) = x^4 + 4x^3 - 18x^2 - 4x + 7$

$f'(x) = 4x^3 + 12x^2 - 36x - 4$

$f''(x) = 12x^2 + 24x - 36$

$0 = 12(x^2 + 2x - 3)$

$0 = 12(x+3)(x-1) = 0$

$x = -3,\ x = 1$

$f''(x)$: $\cup$ + (−4) | −3 | $\cap$ − (0) | 1 | $\cup$ + (2)

$f(x)$ concave up $(-\infty, -3), (1, \infty)$ b/c $f''(x) > 0$

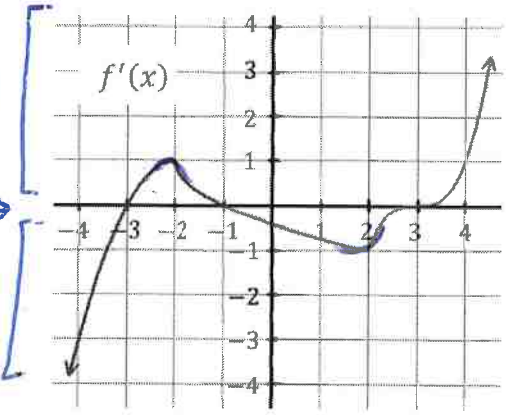
$f(x)$ concave down $(-3, 1)$ b/c $f''(x) < 0$

4) To the right is the graph of $f'(x)$, the **derivative** of a continuous function, f. The domain of f is [-4, 4], **the range of f is [-4,3]**, and $f(-4) = 3$

(Draw separate f '(x) and f ''(x) sign lines)

f '(x) sign line: ↘ ↗ ↘ ↗ at -4, -3, -1, 3, 4

f ''(x) sign line: ∪ + ∩ − ∪ ∪ at -4, -2, 2, 3, 4



Find the following. Justify your answers with "because" statements fo

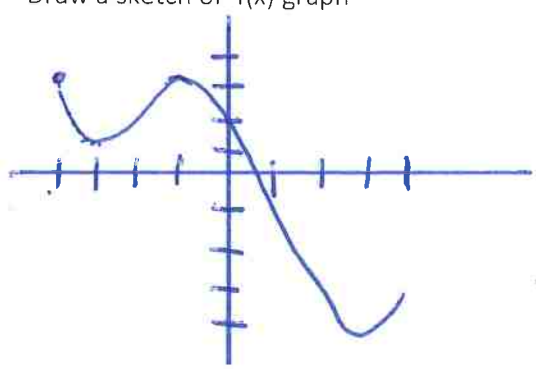
a) interval(s) where f is decreasing $(-4,-3), (-1,3)$ because $f'(x) < 0$

b) interval(s) where f is concave down $(-2,2)$ because $f''(x) < 0$

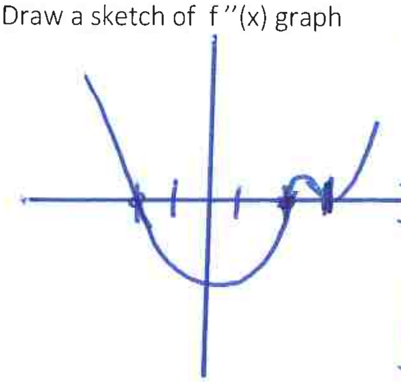
c) x-coordinate of each rel. max $x=-1$ because $f'(x)$ changes from + to −

d) x-coordinate of each pt. of inflection $x=-2, 2$ because $f''(x)$ change sign

e) Draw a sketch of f(x) graph



f) Draw a sketch of f ''(x) graph



5)

Use the 2nd Derivative Test to find x-values of the extrema of $g(x) = 2\cos x - x$ on the interval $(0, 2\pi)$ and justify your answer.

$g'(x) = -2\sin x - 1$

$0 = -2\sin x - 1$

$2\sin x = -1$

$\sin x = -1/2$

$\boxed{x = \frac{11\pi}{6}, \frac{7\pi}{6}}$ Test these candidates for Relative max/min

$g''(x) = -2\cos x$

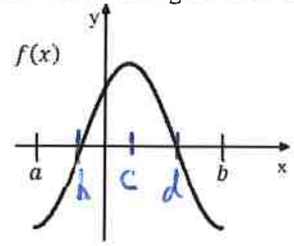
$g''\left(\frac{7\pi}{6}\right) = -2\cos\left(\frac{7\pi}{6}\right) > 0$

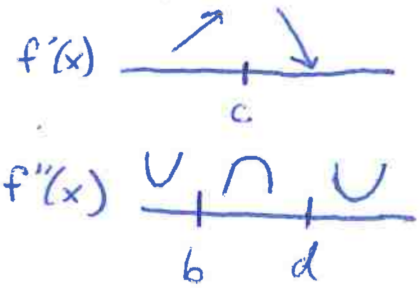
concave up, so relative min at $x = 7\pi/6$

$g''\left(\frac{11\pi}{6}\right) = -2\cos\left(\frac{11\pi}{6}\right) < 0$

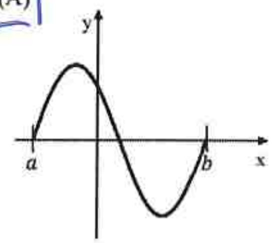
concave down, so relative max at $x = 11\pi/6$

6) The graph of f is shown below. Which of the following could be the graph of the derivative of f?

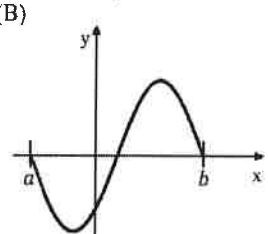




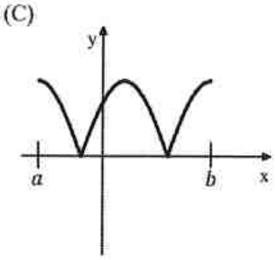
(A)

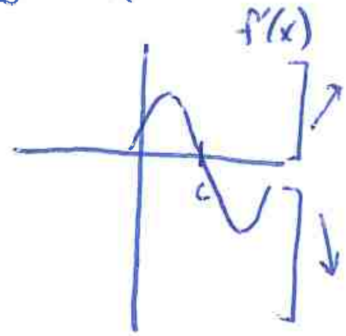


(B)

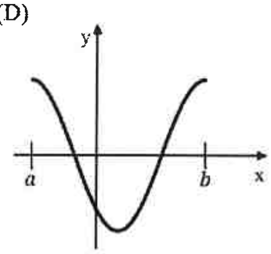


(C)

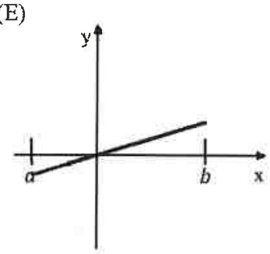




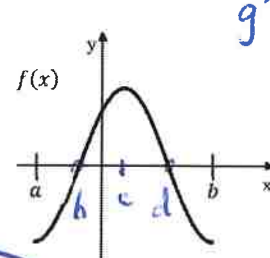
(D)

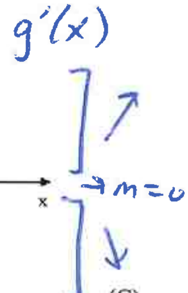


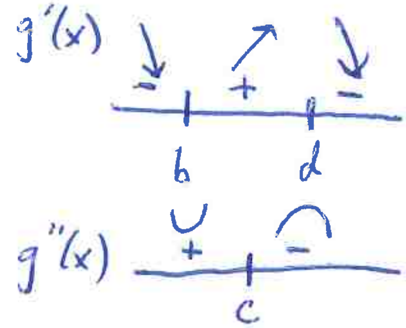
(E)



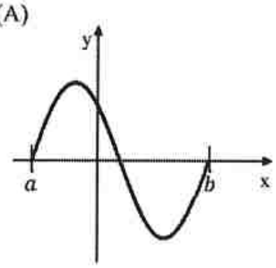
7) **The graph of $f(x)$ *which is the derivative of* $g(x)$*is shown*.** Which of the following could be the graph of g(x)?



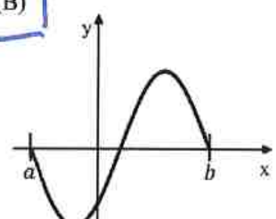




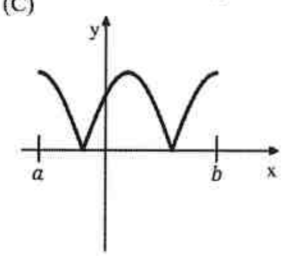
(A)

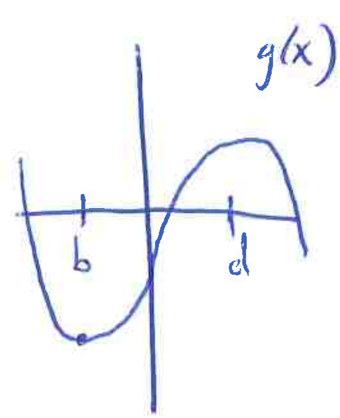


(B)

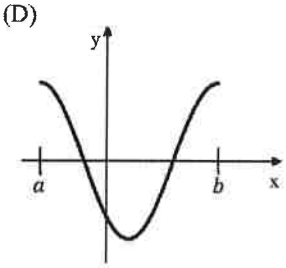


(C)

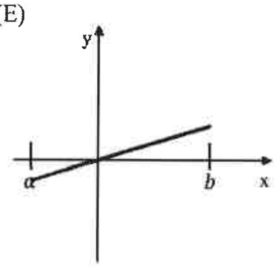




(D)



(E)



8)

The derivative of g is given by $g'(x) = (5-x)x^{-3}$ for $x > 0$. Find all relative extrema and justify your conclusions.

$$g'(x) = \frac{5-x}{x^3}$$

$g'(x)$: $-$ (−1) | 0 | $+$ (1) | 5 | $-$ (6)

critical pts: $5-x=0 \;\Rightarrow\; x=5$ | $x^3=0 \;\Rightarrow\; x=0$

Relative max at $x=5$
b/c $g'(x)$ changes from + to −
(Neither at $x=0$ since VA exist at $x=0$)

9)

Given the function $g(x) = -x^4 + 2x^2 - 1$, find the interval(s) when g is **concave up** and **decreasing** at the same time.

$g'(x) = -4x^3 + 4x$

$0 = -4x(x^2-1)$

$0 = -4x(x+1)(x-1)$

$x = 0, -1, 1$

$g'(x)$: $+$ (−2) | -1 | $-$ (−1/2) | 0 | $+$ (1/2) | 1 | $-$ (2)

$g''(x)$: $-$ (−1) | $-\frac{\sqrt{3}}{3}$ | $+$ (0) | $\frac{\sqrt{3}}{3}$ | $-$ (1)

$g''(x) = -12x^2 + 4$

$0 = -4(3x^2-1)$

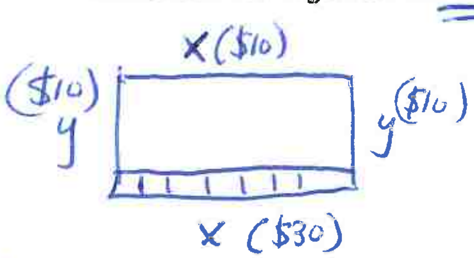
$0 = 3x^2 - 1$

$x^2 = \frac{1}{3}$ $\quad x = \pm\frac{\sqrt{3}}{3}$

$\boxed{g(x) \text{ is concave up and decreasing on interval } \left(-\frac{\sqrt{3}}{3}, 0\right)}$

10)

A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30/foot and on the other 3 sides by a metal fence costing \$10/foot. If the area of the garden is 1000 square feet, find the dimensions of the garden that minimze cost. Round dimensions to 3 decimal places.



Area = xy

$1000 = xy$

$\frac{1000}{x} = y$

* Cost = $40x + 20y$

$C = 40x + 20\left(\frac{1000}{x}\right)$

$C = 40x + 20000x^{-1}$

$C'(x) = 40 - 20000x^{-2}$

$0 = 40 - \frac{20000}{x^2}$

$\frac{20000}{x^2} = 40$

$40x^2 = 20000$

$x^2 = 500$

$\boxed{x = \sqrt{500} = 10\sqrt{5} \text{ ft}}$

$\boxed{y = \frac{1000}{10\sqrt{5}} = \frac{100}{\sqrt{5}} \text{ ft}}$

11. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that it's width is 4 meters and volume is 36 cubic meters. If building the tank cost \$10 per square meter for the base and \$5 per square meter for the sides, what is the cost of the least expensive tank?

$V = 4xy \qquad V = 36\,m^3$

$36 = 4xy \rightarrow \frac{36}{4x} = y \rightarrow \frac{9}{x} = y$

$C = \underbrace{4y}_{\$10} + \underbrace{xy + xy + 4x + 4x}_{\$5}$

$C = 40y + 5(2xy + 8x)$

$C = 40\left(\frac{9}{x}\right) + 10x\left(\frac{9}{x}\right) + 40x$

$C = 360x^{-1} + 90 + 40x$

$C'(x) = -360x^{-2} + 40$

$0 = \frac{-360}{x^2} + 40$

$\frac{360}{x^2} = 40$

$40x^2 = 360$

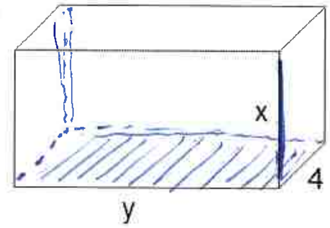
$x^2 = \frac{360}{40}$

$x^2 = 9$

$x = 3$

$C(3) = \frac{360}{3} + 90 + 40(3)$

$= \boxed{\$330}$



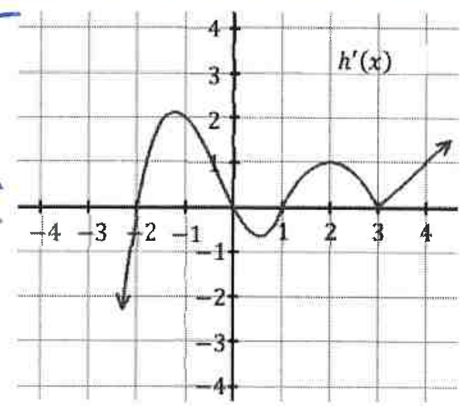
12) To the right is the graph of $h'(x)$. Identify all extrema of $h(x)$. No justification necessary on this problem.

$h'(x)$: $- \mid + \mid - \mid + \mid +$ at $-2,\ 0,\ 1,\ 3$

Rel. min at $x = -2$, $x = 1$

Rel. max at $x = 0$

$m = 0$



13) The graph of the function f is shown in the figure to the right. For which of the following values of x is $f'(x)$ negative and decreasing.

$\boxed{\text{(A) a}}$

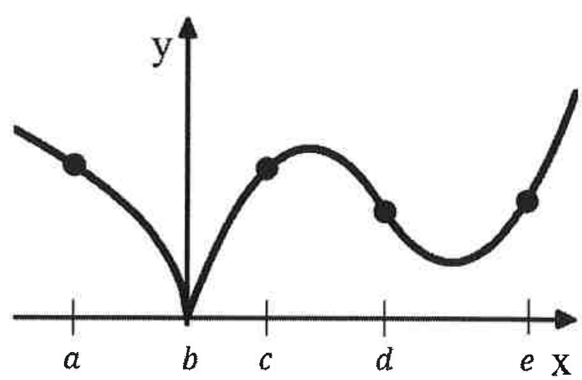
(B) b

(C) c

(D) d

(E) e

Concave down and decreasing at point $\boxed{a}$



14) Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

$\boxed{\text{(A)}}$ The graph of the function has two points of inflection, and the function has one relative extremum.

(B) The graph of the function has one point of inflection, and the function has two relative extrema.

(C) The graph of the function has two points of inflection, and the function has two relative extrema.

(D) The graph of the function has two points of inflection, and the function has three relative extrema.

(E) The function has no relative extremum.

$f'(x) = 4x^3 - 6x^2$

$0 = 2x^2(2x - 3)$

$x = 0,\ x = 3/2$

$f'(x)$: $\searrow$ − | $\searrow$ − | $\nearrow$ +
(−1) 0 (1) 3/2 (2)

$f''(x) = 12x^2 - 12x$

$0 = 12x(x - 1)$

$x = 0,\ 1$

$f''(x)$: $\cup$ + | $\cap$ − | $\cup$ +
(−1) 0 (1/2) 1 (2)

15. Verify whether $f(x) = 3x^2 - 12x + 1$ satisfies Rolle's theorem on the interval [0, 4] and find all numbers c that satisfy $f'(c) = 0$

$f(x)$ continuous on $[0, 4]$, differentiable on $(0, 4)$

A) c = 0
B) c = 1
$\boxed{\text{C)}}$ c = 2
D) c = 4
E) f(x) does not satisfy Rolle's theorem on interval [0, 4]

$f(0) = 1$

$f(4) = 1$

Avg RoC → $\frac{1-1}{4-0} = 0$

$f'(x) = 6x - 12$

* set Avg RoC = Instaneous RoC (derivative)

$0 = 6x - 12$ | $6x - 12 = 0$, $6x = 12$ | $x = 2$ | $\boxed{c = 2}$

16) Which of the following statements is true of the function $f(x) = x^{2/3}$

✓ I. There is a critical point at (0, 0

✓ II. f '(0) and f''(0) are undefined

X III. The curve is concave up over the interval $(0, \infty)$

✓ IV. The curve is concave down over interval $(-\infty, 0)$

A. I and III only
$\boxed{\text{B. I, II, IV only}}$
C. I, II, III
D. I, III, and IV
E. I, II, III, and IV

$f'(x) = \frac{2}{3}x^{-1/3}$

$f'(x) = \frac{2}{3x^{1/3}}$

$3x^{1/3} = 0$

$x^{1/3} = 0$

$x = 0$
(slope undefined at $x = 0$)

$f'(x)$: $\searrow$ − | $\nearrow$ +
(−1) 0 (1)

$f''(x) = \frac{2}{3} \cdot \frac{-1}{3} x^{-4/3}$

$f''(x) = \frac{-2}{9x^{4/3}}$ $x = 0$

$f''(x)$: $\cap$ − | $\cap$ −
(−1) 0 (1)