Name: ______________________________ Period: __________

BC Calculus

Unit 7

Differential Equations and Slope Fields

BC Calculus November 2023 Class Calendar

Monday	Tuesday	Wednesday	Thursday	Friday
Oct 30 Ch. 6 Test FRQ Calculator Review Day 1	Oct 31 7.1 Solving Differential Equations Day 1 HW: 7.1 AP Practice #1-4 all	Nov 1 7.2 - Solving Differential Equations (Day 2) HW: 7.2 AP Practice #1-10 all	2 **Unit 6 Test (Calculator Portion)** **3 FRQs** 1.Derivative Graph 2.Riemann Sums 3. Particle Motion	3 7.3 – Slope Fields HW: **7.3 AP Practice** #17,18, 1-3 all **Unit 7 Cumulative AP Practice** #1,2,5
6 7.1-7.3 Quiz Review	7 **Election Day (No School)**	8 7.1-7.3 Quiz Review	9 8.1 Notes – Area between Graphs HW: pg. 579-582 *3,7,15,21,23,25,27* HW: pg. 579-582 8.1 AP Problems 1-10 all	10 **Chapter 7 Quiz Differential Equations and Slope Fields**
13 8.2a – Volume by Disc Method and Washer Method HW: pg. 592-596 Disc: # 5,7,8 Washer:# 10,47, 49	14 8.2b – Volume by Disc Method and Washer Method HW: 8.2 AP Practice Problems #1-10 all	15 8.4 – Volumes with Cross Sections of various Shapes HW: pg. 610-612 #1,5,7,14, AP Practice #1-8 all	16 8.1-8.4 Quiz Review HW: Ch. 8 Exercise Problems Pg. 633 *1-7 all*	17 8.1-8.4 Quiz Review
20 Thanksgiving Break	21 Thanksgiving Break	22 Thanksgiving Break	23 Thanksgiving Break	24 Thanksgiving Break

6.3 Notes: Differential Equations and Separation of Variables

Separation of Variables: Rearrange equation with y and dy (dependent variable) on the left and the x, dx (independent variable) on the right side of the equation

1) Solve the differential equation $\frac{dy}{dx}=\frac{2x}{y}$

2) Solve $\frac{dy}{dx}=x(1+y)$

3) Find a general solution of $2x + 3yy' = 0$. Then find the particular solution, $y = f(x)$, if the solution passes through the point (1, -2).

(2)

4) Find a general solution to $yy' = 6\cos(\pi x)$. Then find the particular solution, $y = f(x)$, if the function passes through the point (1, 2).

5) Solve $y' = (x + 1)y$

Solving Differential Equations: Summary of Steps

I. Separation of variables

1. Rewrite y' as $\frac{dy}{dx}$ $(or\ \frac{dy}{dt}\ if\ time\ t\ is\ independent\ variable)$

   ***numerator** of the differential fraction (dy) is the **dependent variable.**

   * Denominator of the differential fraction(dx) is the independent variable.

2. Group the dependent variables together on **left side** of equation $(y\ \&\ dy)$

3. Independent variables on the **right side** of equation $(x\ \&\ dx\quad or\quad t\ \&\ dt)$

4. Start by cross-multiplying equation to rearrange all variables in the numerator.

5. Divide terms and/or variables to the other side if the variables are not yet separated.

6. Remember that $\boldsymbol{dy}$ & $\boldsymbol{dx}$ terms need to be in the **numerator location** of their respective sides, never in the denominator once variables are separated.

7. It's ok to have variable(s) $\boldsymbol{x\ or\ y}$ in the denominator after all variables are separated.

8. If parentheses are presented in the differential equation, keep them. Don't expand or distribute. The parentheses are there to help you group the terms that need to stay together.

9. Try to keep the left side of the equation with just the bare minimum terms if possible. $(\boldsymbol{y}, \boldsymbol{dy}\ \&\ any\ terms\ grouped\ with\ y\ in\ parentheses)$. Any coefficient constants keep (or move) to the right side of the equation.

II. Antidifferentiation (Take Integral of both sides)

1. Treat each side as a separate problem and take the appropriate indefinite integral of each side. (Power Rule, other Integral rules, or U-Substitution)

2. We only need to display a "+C" on the right side of the equation.

3. Solve for the "+C" constant

   a) Option 1: Solve for the **C** immediately after it appears. Use the ordered pair given in the problem to solve for C. (my preference is solving for +C if y is raised to a power; example: $\boldsymbol{y^2}, \boldsymbol{y^3}, \boldsymbol{y^{3/2}}$)

   b) Option 2: Wait to solve for **C**. First, clean up the equation by solving for y (isolate y variable) on the left side of the equation (my preference is solving for y first if I see $\boldsymbol{ln(y)}$ on the left side of the equation). Finally, solve for the value of **C** using the given ordered pair.

Calculus AB Chapter 6.2 Notes: Solving Differential Equation Word Problems

1. Direct proportion equation :
2. Inverse (indirect) proportion equation:
3. k is called the ______________________________

Exponential Growth/Decay class examples

1) If the rate of change of y varies directly with the value of y, find the general equation:

2) The rate of increase of the population of a certain city is proportional to the population. If the population in 1930 was 50,000 and in 1960 it was 75,000, what was the expected population in 1990?

(6)

3) The rate of decay of radium is proportional to the amount present at any time. If 60 mg of radium are present now and its half-life is 1690 years, how much radium will be present 100 years from now?

4. In a certain culture where the rate of growth of bacteria is proportional to the amount present, the number triples in 3 hours.
A) If at the end of 12 hours there were 10 million bacteria, how many were present initially?
B) Find the specific exponential growth equation

Note for homework: Newton's law of cooling: the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium

Differential Equations Unit **Slope Fields (Direction Fields) Notes**

Slope Fields: a graphical approach to look at all the solutions of a differential equation. Slope fields consists of short line segments representing slope (steepness) sketched at lots of different points

These line segments are the tangents to a family of solution curves for the differential equation at various points. The tangents show the direction in which the solution curves will follow. Slope fields are useful in sketching solution curves without having to solve a differential equation algebraically.

Steps:

1) Identify the ordered pairs indicated on the graph.
2) Plug in the ordered pairs in the differential equation to find slope
3) Sketch a short line segment representing the slope through the given point
4) Repeat this for all remaining ordered pairs.

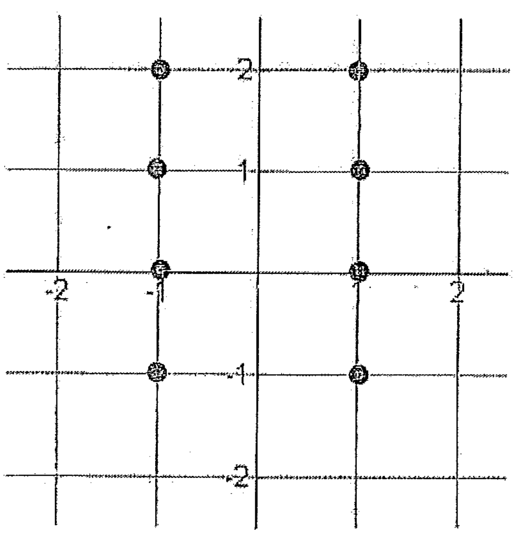
*Use the differential equation to find the individual slope segments, creating the slope field (ex: $\frac{dy}{dx} = x$)

*Use the solution of the differential equation to match with the shape of the slope field: (ex: $y = \frac{1}{2}x^2 + C$)

Example 1: Sketch a slope field for the given differential equation at the indicated eight points.

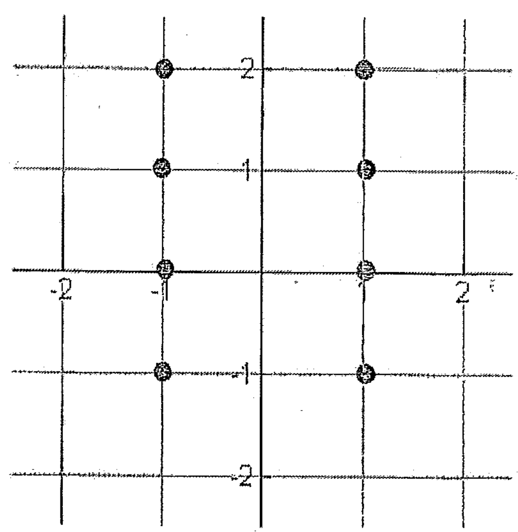
a) $\frac{dy}{dx} = x - 2y$

x								
y								
y'								



b) $\frac{dy}{dx} = \frac{2-x}{y}$

x								
y								
y'								



(8)

Determine the differential equation being graphed by each of the slope fields below. Then sketch a solution that passes through the indicated point.

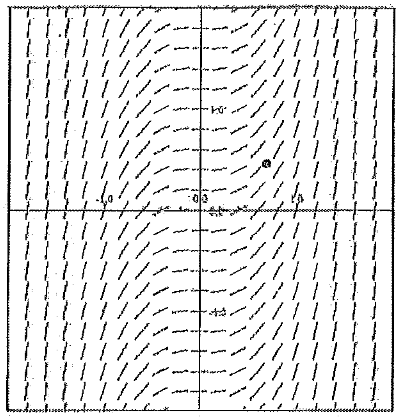
2. a) $\frac{dy}{dx} = x^3$

b) $\frac{dy}{dx} = 3x^2$

c) $\frac{dy}{dx} = 2x + y$

d) $\frac{dy}{dx} = \frac{x}{y}$

e) $\frac{dy}{dx} = \ln x$



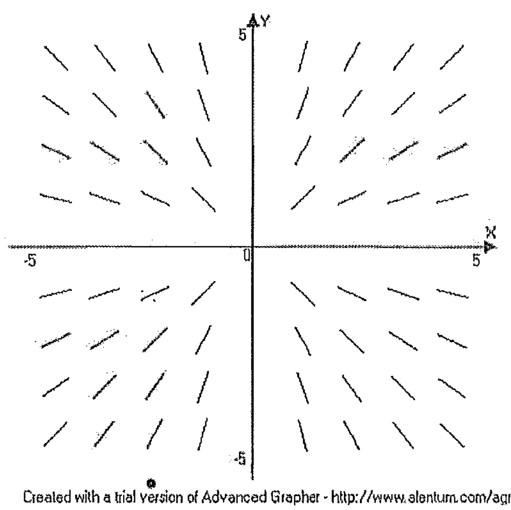
3. a) $\frac{dy}{dx} = x - 2$

b) $\frac{dy}{dx} = x^3$

c) $\frac{dy}{dx} = x - y$

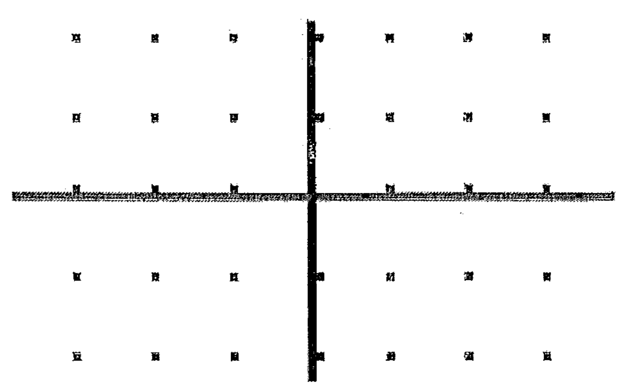
d) $\frac{dy}{dx} = \frac{y}{x}$

e) $\frac{dy}{dx} = e^y$



Sketch slope fields for the following differential equation. Then find the general solution analytically

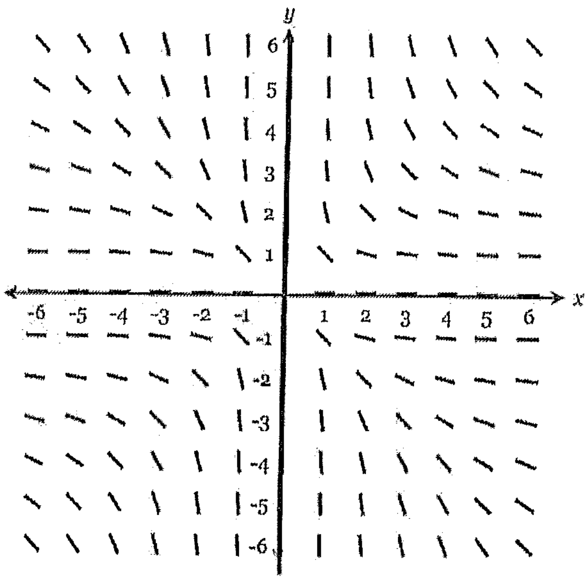
4. $\frac{dy}{dx} = \frac{-x}{y}$



Slope Fields Practice WS

Select the differential equation that matches the given slope field.

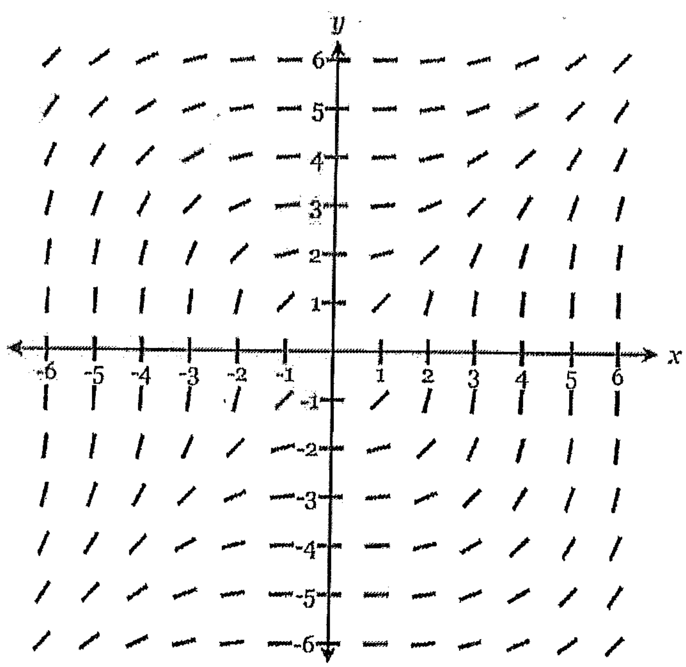
1)



$\frac{dy}{dx} = -xy^2$ $\frac{dy}{dx} = -x^2y$

$\frac{dy}{dx} = \frac{x^2}{y}$ $\frac{dy}{dx} = -\frac{y^2}{x^2}$

2)



$\frac{dy}{dx} = -xy^2$

$\frac{dy}{dx} = \frac{y^2}{x}$

$\frac{dy}{dx} = \frac{x^2}{y^2}$

$\frac{dy}{dx} = \frac{-y^2}{x^2}$

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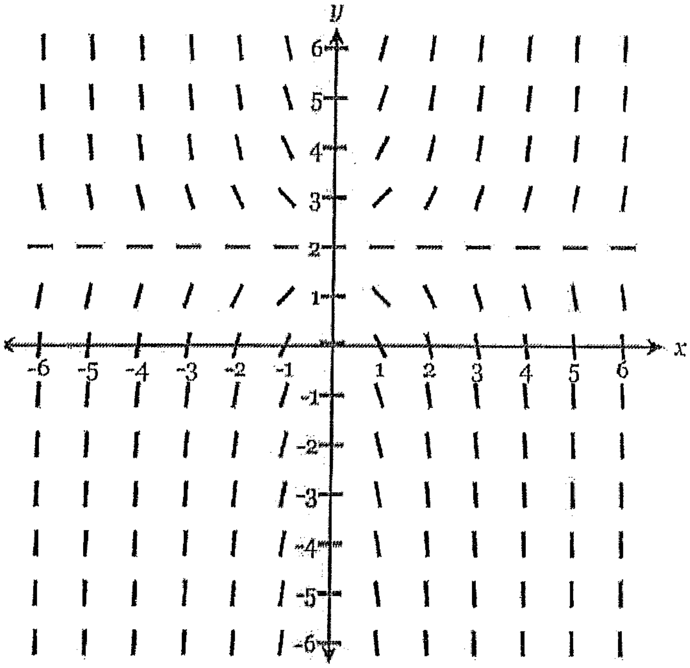
3)



$\frac{dy}{dx} = \frac{x^2}{(y-2)^2}$

$\frac{dy}{dx} = -x^2(y-2)^2$

$\frac{dy}{dx} = x(y-2)$

$\frac{dy}{dx} = -\frac{y-2}{x}$

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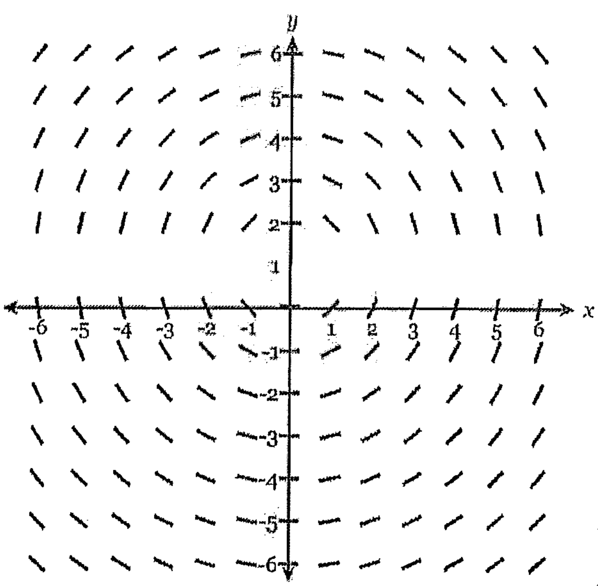
4)



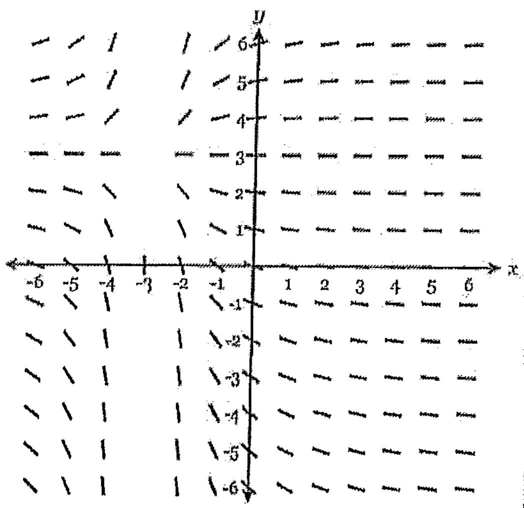
$\frac{dy}{dx} = -\frac{x}{(y-1)^2}$

$\frac{dy}{dx} = -\frac{(y-1)^2}{x}$

$\frac{dy}{dx} = -\frac{x}{y-1}$

$\frac{dy}{dx} = \frac{x^3}{y-1}$

5)



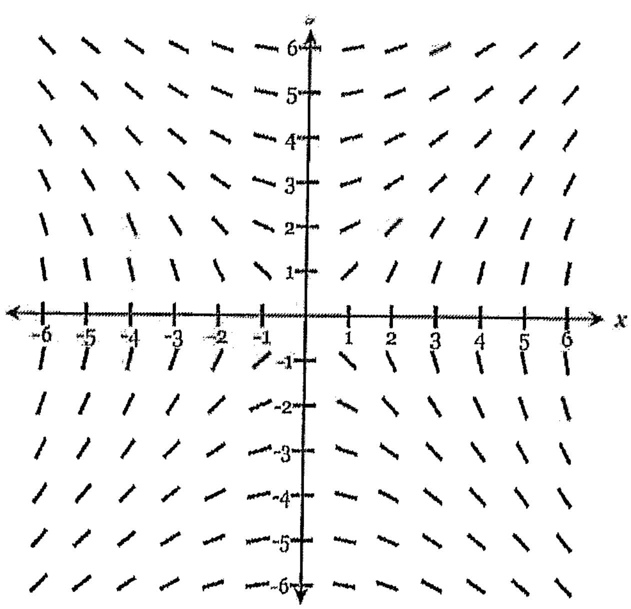
$\frac{dy}{dx} = -(x+3)(y-3)$

$\frac{dy}{dx} = \frac{(y-3)^2}{(x+3)^2}$

$\frac{dy}{dx} = -\frac{x+3}{y-3}$

$\frac{dy}{dx} = \frac{y-3}{(x+3)^2}$

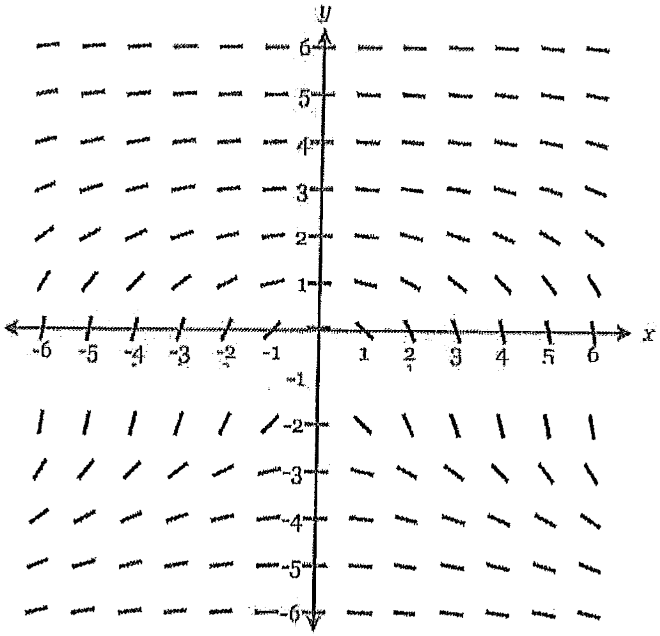
6)



$\frac{dy}{dx} = \frac{x}{y}$ $\frac{dy}{dx} = -xy$

$\frac{dy}{dx} = xy$ $\frac{dy}{dx} = -x^2y$

7)



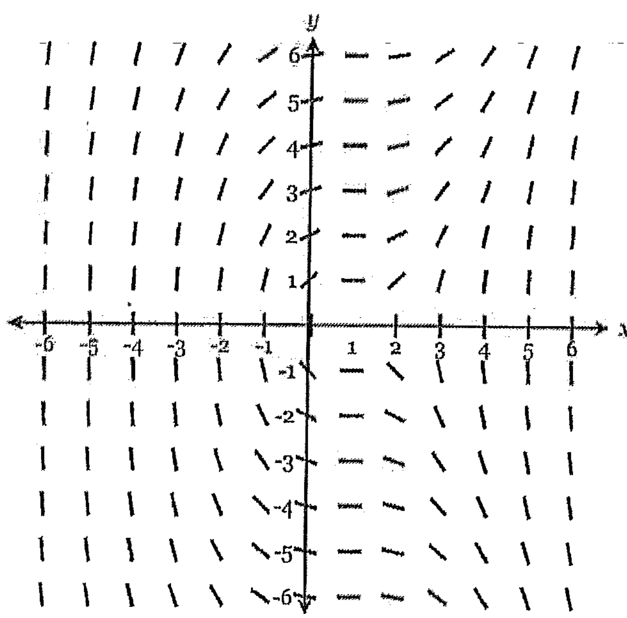
$\frac{dy}{dx} = -\frac{x}{(y+1)^2}$

$\frac{dy}{dx} = x^2(y+1)^2$

$\frac{dy}{dx} = \frac{(y+1)^2}{x}$

$\frac{dy}{dx} = x^2(y+1)$

8)



$\frac{dy}{dx} = \frac{(x-1)^2}{y}$

$\frac{dy}{dx} = \frac{y}{x-1}$

$\frac{dy}{dx} = \frac{x-1}{y^2}$

$\frac{dy}{dx} = -\frac{y}{(x-1)^2}$

Solving Differential Equations Task (part 2)

1)

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

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Solve the below differential equation:

2) $y' - xycos(x^2) = 0$ given $y(0) = e$ a) Find general solution b) Find particular solution

Solving Differential Equations: Additional Practice Problems

3) Solve the Differential Equation: $y \ln x^4 - xy' = 0$

(18)

4) a) Find the general solution b) Find the particular solution

$$yy' - 2e^{3x} = 0 \qquad y(0) = 5$$

Solving Differential Equations Task (Continued)

1) The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\frac{dB}{dt} = \frac{100-B}{5}$$

$$5\,dB = (100-B)\,dt$$

$$\frac{dB}{100-B} = \frac{dt}{5}$$

$$\int \frac{dB}{100-B} = \frac{1}{5}\int dt$$

$u = 100 - B$

$\frac{du}{dB} = -1$

$du = -dB$

$dB = -du$

$$\int \frac{-1\,du}{u} = -\ln|u|$$

$-\ln|100-B| = \frac{1}{5}t + c$

$\ln|100-B| = -\frac{1}{5}t + C$

$e^{\ln|100-B|} = e^{-\frac{1}{5}t + c}$

$|100-B| = e^{-1/5 t} \cdot e^{c}$

$|100-B| = e^{-\frac{1}{5}t} \cdot C$

$|100-B| = Ce^{-\frac{1}{5}t}$

$100 - B = Ce^{-\frac{1}{5}t}$

(t time, B Bird weight) $(0, 20)$

$100 - Ce^{-\frac{1}{5}t} = B$

$B = 100 - Ce^{-\frac{1}{5}t}$ (general equation)

plug in $(0, 20)$

$20 = 100 - Ce^{-\frac{1}{5}(0)}$

$20 = 100 - C$

$C = 80$

$$B = 100 - 80e^{-\frac{1}{5}t}$$

$$B(t) = 100 - 80e^{-\frac{1}{5}t}$$

2) $y' - xy\cos(x^2) = 0 \qquad y(0) = e$

$\frac{dy}{dx} = xy\cos(x^2)$

$dy = xy\cos(x^2)\,dx$

$\frac{dy}{y} = x\cos(x^2)\,dx$

$\int \frac{1}{y}\,dy = \int x\cos(x^2)\,dx$

$u = x^2 \qquad dx = \frac{du}{2x}$

$\frac{du}{dx} = 2x$

$\int x\cos u \cdot \frac{du}{2x}$

$\int \frac{1}{y}\,dy = \frac{1}{2}\int \cos u\,du$

$\ln|y| = \frac{1}{2}\sin u + C$

$\ln|y| = \frac{1}{2}\sin(x^2) + C$

$e^{\ln|y|} = e^{\frac{1}{2}\sin(x^2) + c}$

$|y| = e^{\frac{1}{2}\sin(x^2)} \cdot e^{c}$

$|y| = Ce^{\frac{1}{2}\sin(x^2)}$

$y = Ce^{\frac{1}{2}\sin(x^2)}$ ← general solution

$y = Ce^{\frac{1}{2}\sin(x^2)}$ ← plug in $y(0) = e$

$e = Ce^{\frac{1}{2}\sin(0)^2}$

$e = Ce^{0}$

$e = C$

$y = e \cdot e^{\frac{1}{2}\sin(x^2)}$

$$y = e^{\frac{1}{2}\sin(x^2) + 1}$$

18b

18c

Solving Differential Equations: Additional Practice Problems

3) Solve the Differential Equation: $y\ln x^4 - xy' = 0$

$y\ln x^4 - x\left(\frac{dy}{dx}\right) = 0$

$\frac{-x\,dy}{dx} = \frac{-y\ln x^4}{1}$

$-x\,dy = -y\ln x^4\,dx$

$x\,dy = y\ln x^4\,dx$

$\frac{dy}{y} = \frac{\ln x^4}{x}dx$

$\int \frac{1}{y}dy = \int \frac{4\ln x}{x}dx$ ← $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$, $dx = x\,du$

$4\int \frac{u}{x} x\,du \rightarrow 4\int u\,du$

$4\left(\frac{u^2}{2}\right)$

$\ln|y| = 2u^2 + C$

$\ln|y| = 2(\ln x)^2 + C$

$e^{\ln|y|} = e^{2(\ln x)^2 + C}$

$|y| = e^{2(\ln x)^2} \cdot e^C$

$|y| = e^{2(\ln x)^2} \cdot C$

$\boxed{y = Ce^{2(\ln x)^2}}$

4) a) Find the general solution b) Find the particular solution

$yy' - 2e^{3x} = 0 \qquad y(0) = 5$

$y\left(\frac{dy}{dx}\right) - 2e^{3x} = 0$

$\frac{y\,dy}{dx} = \frac{2e^{3x}}{1}$

$y\,dy = 2e^{3x}dx$

$\int y\,dy = \int 2e^{3x}dx$ ← $u = 3x$, $\frac{du}{dx} = 3$, $dx = \frac{du}{3}$

$2\int e^u \cdot \frac{du}{3}$

$\int y\,dy = \frac{2}{3}\int e^u du$

$\frac{y^2}{2} = \frac{2}{3}e^{3x} + C$ ← solve for C: $y(0) = 5$

$\frac{5^2}{2} = \frac{2}{3}e^{3(0)} + C$

$\frac{25}{2} = \frac{2}{3}(1) + C$

$\frac{25}{2} - \frac{2}{3} = C$

$\frac{75}{6} - \frac{4}{6} = C$

$\frac{71}{6} = C$

$\frac{y^2}{2} = \frac{2}{3}e^{3x} + \frac{71}{6}$

$2\left(\frac{y^2}{2} = \frac{2}{3}e^{3x} + \frac{71}{6}\right)$

$y^2 = \frac{4}{3}e^{3x} + \frac{142}{6}$

$y = \pm\sqrt{\frac{4}{3}e^{3x} + \frac{71}{3}}$

$\boxed{y = \sqrt{\frac{4}{3}e^{3x} + \frac{71}{3}} \text{ since } y(0) = 5}$

Differential Equations Practice WS (1-3)

1)

Given the differential equation $\frac{dy}{dx} = -\frac{2x}{y^2}$, find the particular solution, $y = f(x)$, with the initial condition $f(-1) = 3$.

A) $y = \sqrt{-2x+3}$

B) $y = \sqrt[3]{-3x^2+30}$

C) $y = \sqrt[3]{-3x^2+24}$

D) $y = \sqrt{-2x+7}$

E) $y = \sqrt{-3x^2-10}$

2) Given the differential equation $\frac{y'}{3-x} = 6y$, find the particular solution, $y = f(x)$, with the initial condition f(0) = 2

A) $y = \sqrt{-\frac{3}{2}x^2 + x + 2}$

B) $y = \sqrt{-3x^2+36x+4}$

C) $y = \ln|18x - 3x^2| + 2$

D) $y = e^{18x-3x^2} + 2$

E) $y = 2e^{18x-3x^2}$

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3)

Given the differential equation $\frac{dy}{dx} = \frac{2x-1}{y}$, find the particular solution, $y = f(x)$, with the initial condition $f(-3) = 6$.

4)

What is the particular solution to the differential equation $\frac{dy}{dx} = x^2 y$ with the initial condition $y(3) = e$?

5)

Given the differential equation, $ww' = t^2 sec^2(2t^3)$, find the particular solution, $w = f(t)$, with the initial condition $w(0) = -4$.

6)

Given the differential equation, $y'xlnx - y = 0$, find the particular solution, $y = f(x)$, with the initial condition $f(e) = e$

6.3/6.2 Solving Differential Equations Formative Check-in Name: Key Period:____

1)

Given the differential equation $\frac{dy}{dx} = -\frac{2x}{y^2}$, find the particular solution, $y = f(x)$, with the initial condition $f(-1) = 3$.

1 A) $y = \sqrt{-2x+3}$

5 B) $y = \sqrt[3]{-3x^2+30}$

4 C) $y = \sqrt[3]{-3x^2+24}$

1 D) $y = \sqrt{-2x+7}$

2 E) $y = \sqrt{-3x^2-10}$

$y^2 dy = -2x\,dx$

$\int y^2 dy = \int -2x\,dx$

$\frac{y^3}{3} = \frac{-2x^2}{2} + C$ ← $(-1, 3)$

$\frac{(3)^3}{3} = \frac{-2(-1)^2}{2} + C$

$9 = -1 + C$

$10 = C$

$\frac{y^3}{3} = -x^2 + 10$

$\left(\frac{y^3}{3} = -x^2 + 10\right)3$

$y^3 = -3x^2 + 30$

$y = \sqrt[3]{-3x^2+30}$

6.3/6.2 Solving Differential Equations Formative Check-in Name: ____________ Period:____

1)

Given the differential equation $\frac{dy}{dx} = -\frac{2x}{y^2}$, find the particular solution, $y = f(x)$, with the initial condition $f(-1) = 3$.

A) $y = \sqrt{-2x+3}$

B) $y = \sqrt[3]{-3x^2+30}$

C) $y = \sqrt[3]{-3x^2+24}$

D) $y = \sqrt{-2x+7}$

E) $y = \sqrt{-3x^2-10}$

2) Given the differential equation $\frac{y'}{3-x} = 6y$, find the particular solution, $y = f(x)$, with the initial condition f(0) = 2

2 A) $y = \sqrt{-\frac{3}{2}x^2 + x + 2}$

2 B) $y = \sqrt{-3x^2 + 36x + 4}$

1 C) $y = \ln|18x - 3x^2| + 2$

4 D) $y = e^{18x-3x^2} + 2$

5 E) $y = 2e^{18x-3x^2}$

$\frac{y'}{3-x} = 6y$

$y' = 6y(3-x)$

$\frac{dy}{dx} = \frac{6y(3-x)}{1}$

$dy = 6y(3-x)dx$

$\frac{dy}{y} = 6(3-x)dx$

$\int \frac{1}{y} dy = \int 18 - 6x\,dx$

$\ln|y| = 18x - \frac{6x^2}{2} + C$

$\ln|y| = 18x - 3x^2 + C$

$e^{\ln|y|} = e^{18x-3x^2+C}$

$|y| = e^{18x-3x^2} \cdot e^C$

$|y| = e^{18x-3x^2} \cdot C$

$|y| = Ce^{18x-3x^2}$

$y = Ce^{18x-3x^2}$ ← $(0, 2)$

$2 = Ce^{18(0)-3(0)^2}$

$2 = Ce^0$

$2 = C$

$y = 2e^{18x-3x^2}$

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2) Given the differential equation $\frac{y'}{3-x} = 6y$, find the particular solution, $y = f(x)$, with the initial condition f(0) = 2

A) $y = \sqrt{-\frac{3}{2}x^2 + x + 2}$

B) $y = \sqrt{-3x^2 + 36x + 4}$

C) $y = \ln|18x - 3x^2| + 2$

D) $y = e^{18x-3x^2} + 2$

E) $y = 2e^{18x-3x^2}$

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6.3/6.2 Differential Equations Formative Check Part 2 Name: ____________ Period: ____ Key

1)

Given the differential equation $\frac{dy}{dx} = \frac{2x-1}{y}$, find the particular solution, $y = f(x)$, with the initial condition $f(-3) = 6$.

2)

What is the particular solution to the differential equation $\frac{dy}{dx} = x^2y$ with the initial condition $y(3) = e$? Key

6.3/6.2 Differential Equations Formative Check Part 2 Name: ____________ Period: ____ Key

1)

Given the differential equation $\frac{dy}{dx} = \frac{2x-1}{y}$, find the particular solution, $y = f(x)$, with the initial condition $f(-3) = 6$.

$y\,dy = 2x-1\,dx$

$\int y\,dy = \int 2x-1\,dx$

$\frac{y^2}{2} = \frac{2x^2}{2} - 1x + C$

$\frac{y^2}{2} = x^2 - x + C$ ← plug in (-3,6)

$\frac{6^2}{2} = (-3)^2 - (-3) + C$

$18 = 9 + 3 + C$

$18 = 12 + C$

$6 = C$

$\frac{y^2}{2} = x^2 - x + C$

$2\left(\frac{y^2}{2} = x^2 - X + 6\right)$

$y^2 = 2x^2 - 2x + 12$

$y = \sqrt{2x^2 - 2x + 12}$

2)

What is the particular solution to the differential equation $\frac{dy}{dx} = x^2y$ with the initial condition $y(3) = e$? Key

$\frac{dy}{dx} = \frac{x^2y}{1}$

$dy = x^2y\,dx$

$\frac{dy}{y} = x^2dx$

$\int \frac{1}{y}dy = \int x^2dx$

$\ln|y| = \frac{x^3}{3} + C$

$e^{\ln|y|} = e^{x^3/3 + C}$

$|y| = e^{x^3/3} \cdot e^C$

$|y| = e^{x^3/3} \cdot C$

$|y| = Ce^{x^3/3}$

$y = Ce^{x^3/3}$ ← plug in (3,e)

$e = Ce^{3^3/3}$

$e = Ce^9$

$\frac{e}{e^9} = C$

$\frac{1}{e^8} = C$

$y = \frac{1}{e^8} \cdot e^{x^3/3}$

$y = e^{-8} \cdot e^{x^3/3}$

$y = e^{x^3/3 - 8}$

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6.3/6.2 Solving Differential Equations Mini WS #3 Name: Key Period:____

1. Given the differential equation, $ww' = t^2sec^2(2t^3)$, find the particular solution, $w = f(t)$, with the initial condition $w(0) = -4$.

$\frac{w \cdot dw}{dt} = \frac{t^2 sec^2(2t^3)}{1}$

$w\,dw = t^2 sec^2(2t^3)\,dt$

$\int w\,dw = \int t^2 sec^2(2t^3)\,dt$

$u = 2t^3 \quad dt = \frac{du}{6t^2}$

$\frac{du}{dt} = 6t^2$

$= \int t^2 \cdot sec^2 u \cdot \frac{du}{6t^2}$

$\int w\,dw = \frac{1}{6}\int sec^2 u\,du$

$\frac{w^2}{2} = \frac{1}{6}\tan u + C$

$\frac{w^2}{2} = \frac{1}{6}\tan(2t^3) + C$ ← solve for C, plug in (0,−4)

$\frac{(-4)^2}{2} = \frac{1}{6}\tan(2(0)^3) + C$

$8 = 0 + C$

$8 = C$

$\left(\frac{w^2}{2} = \frac{1}{6}\tan(2t^3) + 8\right)2$

$w^2 = \frac{2}{6}\tan(2t^3) + 16$

$\boxed{w = -\sqrt{\frac{1}{3}\tan(2t^3) + 16}}$

2. Given the differential equation, $y'x\ln x - y = 0$, find the particular solution, $y = f(x)$, with the initial condition $f(e) = e$ Key

$\frac{dy}{dx} \cdot x\ln x = \frac{y}{1}$

$x\ln x\,dy = y\,dx$

$\frac{dy}{y} = \frac{dx}{x\ln x}$

$\int \frac{dy}{y} = \int \frac{dx}{x\ln x}$

$u = \ln x \quad dx = x\,du$

$\frac{du}{dx} = \frac{1}{x}$

$\int \frac{dy}{y} = \int \frac{x\,du}{x \cdot u} \rightarrow \int \frac{1}{u}du$

$\ln|y| = \ln|u| + c$

$\ln|y| = \ln|\ln x| + c$

$e^{\ln|y|} = e^{\ln|\ln x| + c}$

$|y| = e^{\ln|\ln x|} \cdot e^c$

$|y| = |\ln x| \cdot C$

$y = C\ln x$ ← plug in (e,e)

$e = C\ln e$

$e = C(1)$

$e = C$

$\boxed{y = e\ln x}$

6.3/6.2 Solving Differential Equations Mini WS #3 Name:____________ Period:____

1. Given the differential equation, $ww' = t^2sec^2(2t^3)$, find the particular solution, $w = f(t)$, with the initial condition $w(0) = -4$.

2. Given the differential equation, $y'x\ln x - y = 0$, find the particular solution, $y = f(x)$, with the initial condition $f(e) = e$

7.1 AP Practice Problems

PAGE 540 1. Identify the differential equation for which $y = \pi x + \sin^3 x$ is a solution.

(A) $\frac{dy}{dx} = \pi x + 3\sin^2 x \cos x$

(B) $\frac{dy}{dx} = \pi + 3\sin^2 x \cos x$

(C) $\frac{dy}{dx} = -3\sin^2 x \cos x$

(D) $\frac{dy}{dx} = \pi + 3(\sin x)^2$

PAGE 540 2. Identify the differential equation for which $y = e^{3x-4}$ is a solution.

(A) $y' = 3e^{3x-4}$ (B) $y' = 3xe^{3x-4}$

(C) $y' = \frac{1}{3}e^{3x-4}$ (D) $y' = \frac{1}{3x-4}e^{3x-4}$

PAGE 541 3. The general solution to the differential equation $y' = (x-3)^2(2x+1)$ is

(A) $y = \left(\frac{x-3}{3}\right)^3(2x+4) + (x-3)\left(\frac{2x+1}{2}\right)^2 + C$

(B) $y = 6x^2 - 22x + 12 + C$

(C) $y = 2x^4 - 11x^3 + 12x^2 + C$

(D) $y = \frac{1}{2}x^4 - \frac{11}{3}x^3 + 6x^2 + 9x + C$

(26)

PAGE 542 4. The particular solution of the differential equation $\frac{dy}{dx} = x\sqrt[3]{x^2 - 1}$ with the initial condition, if $x = 3$, then $y = 2$ is

(A) $y = \frac{3}{4}(x^2 - 1)^{4/3} - 10$ (B) $y = \frac{3}{8}(x^2 - 1)^{4/3} - 4$

(C) $y = \frac{3}{8}(x^2 - 1)^{4/3} + 6$ (D) $y = \frac{3}{4}(x^2 - 1)^{4/3} + 14$

7.2 AP Practice Problems

1. Find the solution of the differential equation $\frac{dy}{dx} = \frac{\cos x}{3y^2}$, with the boundary condition $y\left(\frac{\pi}{6}\right) = 1$.

(A) $y^3 = \sin x - \frac{1}{2}$ (B) $y = \sin x + \frac{1}{2}$

(C) $y^3 = \sin x + \frac{1}{2}$ (D) $y^3 = \sin x + \frac{\sqrt{3}}{2}$

2. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x}{y}$, with the initial condition $y(0) = 1$?

(A) $y = \sqrt{x^2 + 1}$ (B) $y = x^2 + 1$

(C) $y = \pm\sqrt{x^2 + 1}$ (D) $y = -\sqrt{x^2 + 1}$

3. Suppose $\frac{dy}{dx} = e^y \cos x$, and $y = 0$ when $x = \pi$.

Then evaluate y when $x = \frac{\pi}{6}$.

(A) $\ln \frac{1}{2}$ (B) $\ln 2$ (C) $\ln\left(1 - \frac{\sqrt{3}}{2}\right)^{-1}$ (D) $\frac{1}{2}$

4. Solve $\frac{dy}{dx} = x^3 y$. Then y equals

(A) $\frac{4}{Cx^4}$ (B) $\frac{x^4}{4} + C$ (C) Ce^{3x^2} (D) $Ce^{x^4/4}$

5. If $\frac{dy}{dx} = 5y^2$ and $y = 1$ when $x = 3$, then find y when $x = 0$.

(A) $-\frac{1}{6}$ (B) $-\frac{1}{16}$ (C) $\frac{1}{16}$ (D) $\frac{1}{6}$

6. If $\frac{dy}{dx} = \frac{y}{1+x^2}$ and $y = 1$ if $x = -1$, then y equals

(A) $\frac{\pi}{4} e^{\tan^{-1} x}$ (B) $e^{\tan^{-1} x} + \frac{\pi}{4}$

(C) $e^{\tan^{-1} x} + e^{\pi/4}$ (D) $e^{\tan^{-1} x + \pi/4}$

7. A population of insects increases according to the uninhibited growth equation $\frac{dP}{dt} = kP$, where k is a constant and t is time in days. If the population doubles every 12 days, then k equals

(A) $\frac{\ln 2}{12}$ (B) $\frac{(\ln 2)^2}{\ln 12}$

(C) $(\ln 2) \ln 12$ (D) $\log_2 12$

8. Suppose $\frac{dA}{dt} = k(100 - A)$, where $k > 0$ is a constant and $A < 100$. If $A = A_0$ when $t = 0$, then

(A) $A = A_0 e^{kt}$ (B) $A = (100 - A_0)e^{-kt}$

(C) $A = 100 - (100 - A_0)e^{-kt}$ (D) $A = (100 - A_0)e^{-100kt}$

9. A colony of bacteria is growing at a rate $\frac{dB}{dt} = 6e^{3t/4}$ grams per hour. If initially there are 8 grams of bacteria in the colony, how many grams will be present in 12 hours?

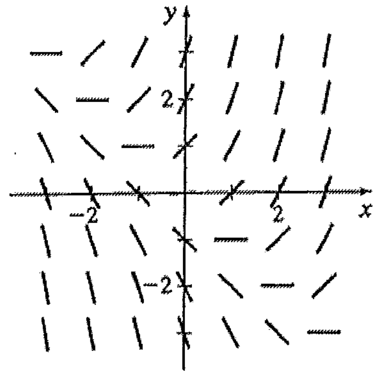
(A) 12 g (B) 72 g (C) $6e^9$g (D) $8e^9$g

10. An apple pie is baked to a temperature of 400°F then placed on a rack to cool in a room with a constant temperature of 70°F. After 20 min the temperature of the pie is 300°F. To the nearest degree, what is the temperature of the pie after 60 min?

(A) 86°F (B) 100°F (C) 182°F (D) 190°F

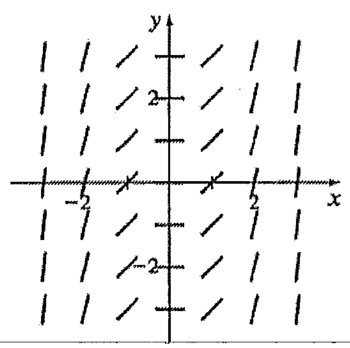
7.3 AP Practice and Exercise Problems (#17 and 18)

17. Which of the following differential equations could have the slope field shown below?



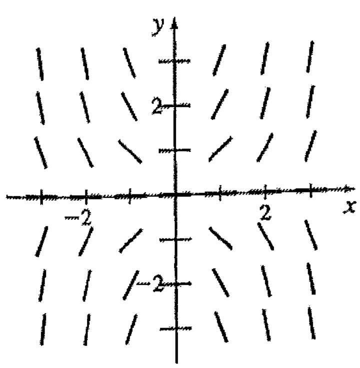
(a) $\frac{dy}{dx} = -x$ (b) $\frac{dy}{dx} = x + y$ (c) $\frac{dy}{dx} = x$ (d) $\frac{dy}{dx} = x - y$

18. Which of the following differential equations could have the slope field shown below?



(a) $\frac{dy}{dx} = -x$ (b) $\frac{dy}{dx} = x^2$ (c) $\frac{dy}{dx} = 2x + 1$ (d) $\frac{dy}{dx} = -x^2$

1. The slope field shown in the figure represents the solution to which differential equation?



(A) $\frac{dy}{dx} = x + y$ (B) $\frac{dy}{dx} = xy$

(C) $\frac{dy}{dx} = x - y$ (D) $\frac{dy}{dx} = \frac{x}{y}$

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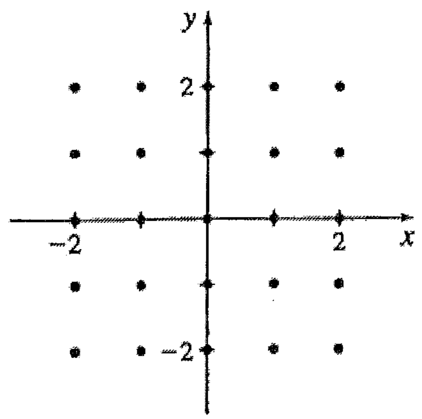
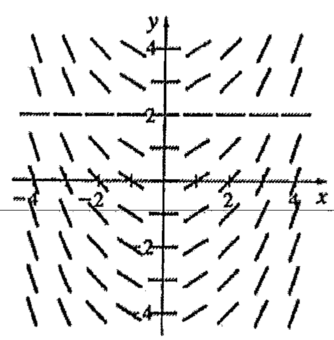
2. (a) Draw a slope field for the differential equation $\frac{dy}{dx} = 2x - y$, using the grid below.

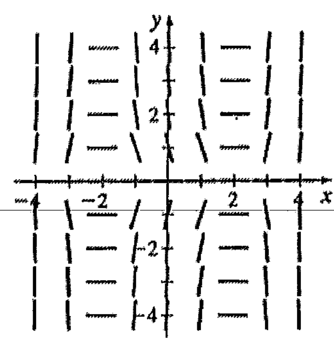
(b) Use the slope field in (a) to draw the solution of the differential equation that satisfies the boundary condition $y = 0$ when $x = 0$.



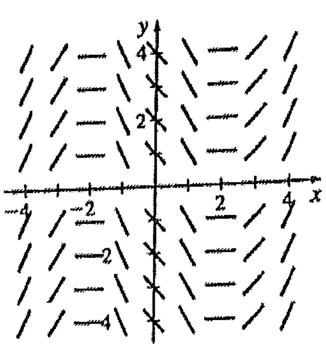
3. Which of the following represents the slope field of $\frac{dy}{dx} = x^2y - 4y$?



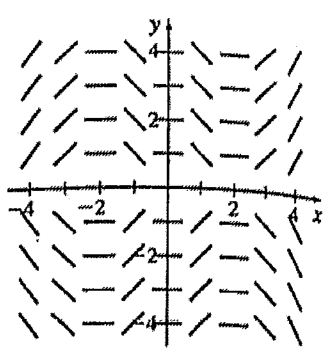
(A)



(B)



(C)



(D)

Unit 7 AP Cumulative Review Problems

1. If $\frac{dy}{dx} = 2(1+y^2)x$, then

   (A) $y = x^2 + C$ (B) $\tan^{-1} y = x^2 + C$

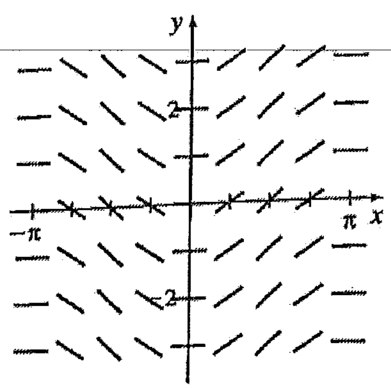
   (C) $y = Ce^{x^2}$ (D) $y = \sqrt{e^{x^2}} + C$

2. If at every point (x, y) on the graph of a function f, the slope of the tangent line is given by $y = 3 - 4x$ and if the point $(2, 3)$ is on the graph of f, then

   (A) $f(x) = -5x + 7$ (B) $f(x) = -2x^2 + 3x - 11$

   (C) $f(x) = -2x^2 + 3x$ (D) $f(x) = -2x^2 + 3x + 5$

5. The slope field shown in the figure represents the solutions to which differential equation?



   (A) $\frac{dy}{dx} = -x^4$ (B) $\frac{dy}{dx} = \cos x$

   (C) $\frac{dy}{dx} = \sin x$ (D) $\frac{dy}{dx} = x^3$