Key

Unit 7 Differential Equations and Slope Fields Quiz Review WS 2

1) Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

(a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point (0, 2), and sketch the solution curve that passes through the point (1, 0).

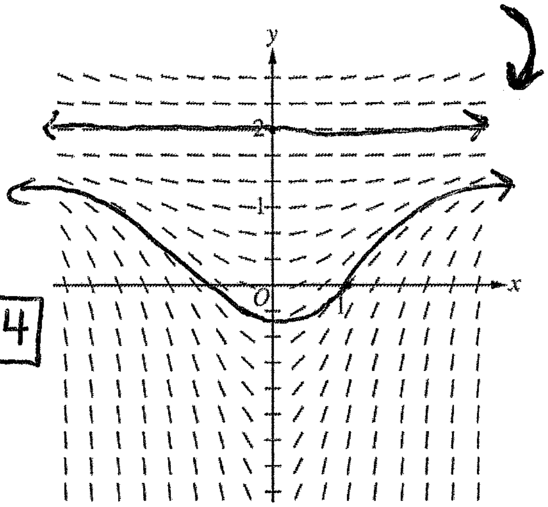
point: (1,0)

b) slope: $\left.\frac{dy}{dx}\right|_{(1,0)} = \frac{1}{3}(1)(0-2)^2 = \frac{4}{3}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{4}{3}(x-1)$

$y(0.7) = \frac{4}{3}(0.7-1) = \boxed{-0.4}$



(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$\frac{dy}{dx} = \frac{x(y-2)^2}{3}$

$3dy = x(y-2)^2 dx$

$\frac{dy}{(y-2)^2} = \frac{1}{3}x\,dx$

$\int \frac{1}{(y-2)^2}dy = \int \frac{1}{3}x\,dx$

$\int (y-2)^{-2}dy = \int \frac{1}{3}x\,dx$

$u = y-2$, $\frac{du}{dy} = 1$

$\int u^{-2}du$

$\frac{u^{-1}}{-1}$

$\downarrow$

$\frac{-1}{(y-2)} = \frac{1}{3}\left(\frac{x^2}{2}\right) + c$

$\frac{-1}{y-2} = \frac{x^2}{6} + c$ ← plug in (1,0)

$\frac{-1}{0-2} = \frac{1}{6} + c$

$\frac{1}{2} - \frac{1}{6} = c \quad c = \frac{1}{3}$

$\frac{-1}{y-2} = \frac{x^2}{6} + \frac{1}{3}$

$\frac{-1}{y-2} = \frac{x^2+2}{6}$

$(y-2)(x^2+2) = -6$

$y - 2 = \frac{-6}{x^2+2}$

$\boxed{y = \frac{-6}{x^2+2} + 2}$

2) A petri dish contains 100 bacteria, and the number N of bacteria is increasing according to the equation $\frac{dN}{dt} = kN$, where k is a constant and t is measured in hours. At time $t = 3$, there are 181 bacteria. Based on this information, what is the doubling time for the bacteria?

$N = Ce^{kt}$

(time, N bacteria)
(0, 100)
(3, 181)
(__, 200)

$100 = Ce^{k(0)}$

$100 = C$

$N = 100e^{kt}$

$181 = 100e^{k(3)}$

$1.81 = e^{3k}$

$\ln 1.81 = \ln e^{3k}$

$\ln 1.81 = 3k \cancel{\ln e}$

$\frac{1}{3}\ln 1.81 = k$

$N = 100e^{\frac{1}{3}\ln(1.81)t}$

$200 = 100e^{\frac{1}{3}\ln(1.81)t}$

$2 = e^{\frac{1}{3}\ln(1.81)t}$

$\ln 2 = \ln e^{\frac{1}{3}\ln(1.81)t}$

$\ln 2 = \frac{1}{3}\ln(1.81)t$

$\frac{\ln 2}{\frac{1}{3}\ln(1.81)} = t$

$\boxed{t \approx 3.505 \text{ hrs}}$

3) The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$\frac{dB}{dt} = \frac{100-B}{5}$

$5\,dB = (100-B)\,dt$

$\frac{dB}{100-B} = \frac{dt}{5}$

$\int \frac{dB}{100-B} = \frac{1}{5}\int dt$

$u = 100 - B$

$\frac{du}{dB} = -1$

$du = -dB$

$dB = -du$

$\int \frac{-1\,du}{u}$

$= -\ln|u|$

$-\ln|100-B| = \frac{1}{5}t + c$

(t time, B Bird weight) (0, 20)

$\ln|100-B| = -\frac{1}{5}t + C$

$e^{\ln|100-B|} = e^{-\frac{1}{5}t + c}$

$|100-B| = e^{-1/5 t} \cdot e^{c}$

$|100-B| = e^{-\frac{1}{5}t} \cdot C$

$|100-B| = Ce^{-\frac{1}{5}t}$

$100 - B = Ce^{-\frac{1}{5}t}$

$100 - Ce^{-\frac{1}{5}t} = B$

$B = 100 - Ce^{-\frac{1}{5}t}$ (general equation) ← plug in (0,20)

$20 = 100 - Ce^{-\frac{1}{5}(0)}$

$20 = 100 - C$

$C = 80$

$B = 100 - 80e^{-\frac{1}{5}t}$

$\boxed{B(t) = 100 - 80e^{-\frac{1}{5}t}}$

4) For what value of k, if any, is $y = e^{-2x} + ke^{5x}$ a solution to the differential equation $\boxed{2y' + y'' = 15e^{5x}}$?

$y' = e^{-2x}(-2) + ke^{5x}(5)$

$y'' = -2e^{-2x}(-2) + 5ke^{5x}(5)$

$2\left[-2e^{-2x} + 5ke^{5x}\right] + \left[4e^{-2x} + 25ke^{5x}\right] = 15e^{5x}$

$\cancel{-4e^{-2x}} + 10ke^{5x} + \cancel{4e^{-2x}} + 25ke^{5x} = 15e^{5x}$

$35ke^{5x} = 15e^{5x}$

$k = \frac{15e^{5x}}{35e^{5x}} \rightarrow k = \frac{15}{35} \rightarrow \frac{3}{7}$ $\boxed{k = \frac{3}{7}}$

6.3/6.2 Solving Differential Equations Mini WS #3 Name: Key Period:____

5.) Given the differential equation, $ww' = t^2 sec^2(2t^3)$, find the particular solution, $w = f(t)$, with the initial condition $w(0) = -4$.

$$\frac{w \cdot dw}{dt} = \frac{t^2 \sec^2(2t^3)}{1}$$

$$w\,dw = t^2 \sec^2(2t^3)\,dt$$

$$\int w\,dw = \int t^2 \sec^2(2t^3)\,dt$$

$u = 2t^3 \quad dt = \frac{du}{6t^2}$

$\frac{du}{dt} = 6t^2$

$$= \int \cancel{t^2} \cdot \sec^2 u \cdot \frac{du}{6\cancel{t^2}}$$

$$\int w\,dw = \frac{1}{6}\int \sec^2 u\,du$$

$$\frac{w^2}{2} = \frac{1}{6}\tan u + C$$

$$\frac{w^2}{2} = \frac{1}{6}\tan(2t^3) + C$$

solve for C
plug in (0,-4)

$$\frac{(-4)^2}{2} = \frac{1}{6}\tan(2(0)^3) + C$$

$$8 = 0 + C$$

$$8 = C$$

$$\left(\frac{w^2}{2} = \frac{1}{6}\tan(2t^3) + 8\right)2$$

$$w^2 = \frac{2}{6}\tan(2t^3) + 16$$

$$\boxed{w = -\sqrt{\frac{1}{3}\tan(2t^3) + 16}}$$

6.) Given the differential equation, $y'x\ln x - y = 0$, find the particular solution, $y = f(x)$, with the initial condition $f(e) = e$ $\boxed{\text{Key}}$

$$\frac{dy\, x\ln x}{dx} = \frac{y}{1}$$

$$x\ln x\,dy = y\,dx$$

$$\frac{dy}{y} = \frac{dx}{x\ln x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x\ln x}$$

$u = \ln x \quad dx = x\,du$

$\frac{du}{dx} = \frac{1}{x}$

$$\int \frac{dy}{y} = \int \frac{\cancel{x}\,du}{\cancel{x} \cdot u} \rightarrow \int \frac{1}{u}\,du$$

$$\ln|y| = \ln|u| + c$$

$$\ln|y| = \ln|\ln x| + c$$

$$e^{\ln|y|} = e^{\ln|\ln x| + c}$$

$$|y| = e^{\ln|\ln x|} \cdot e^{c}$$

$$|y| = |\ln x| \cdot C$$

$$y = C\ln x$$

plug in (e,e)

$$e = C\ln e$$

$$e = C(1)$$

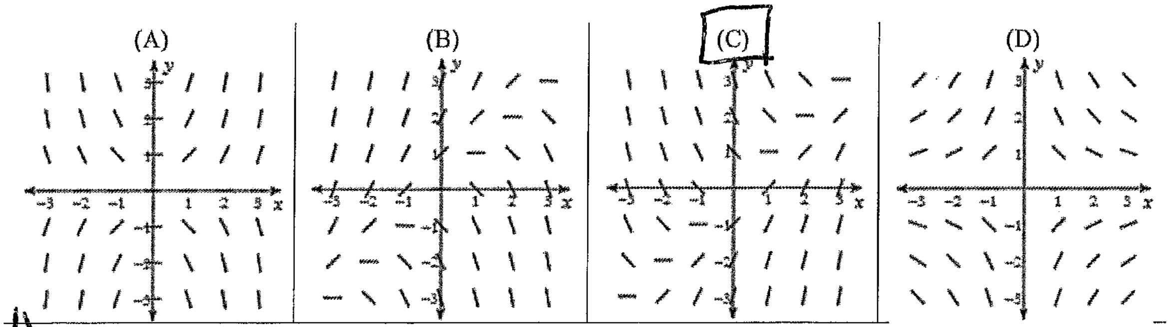
$$e = C$$

$$\boxed{y = e\ln x}$$

7) Slope Fields Practice:

Match the slope field with the differential equation.

a) $\frac{dy}{dx} = x - y$



b)

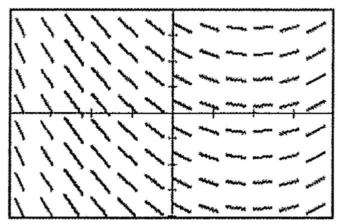
(A) $\frac{dy}{dx} = (x-2)^2$

$\boxed{\text{(B)}}$ $\frac{dy}{dx} = 0.5x - 1$

(C) $\frac{dy}{dx} = x - y$

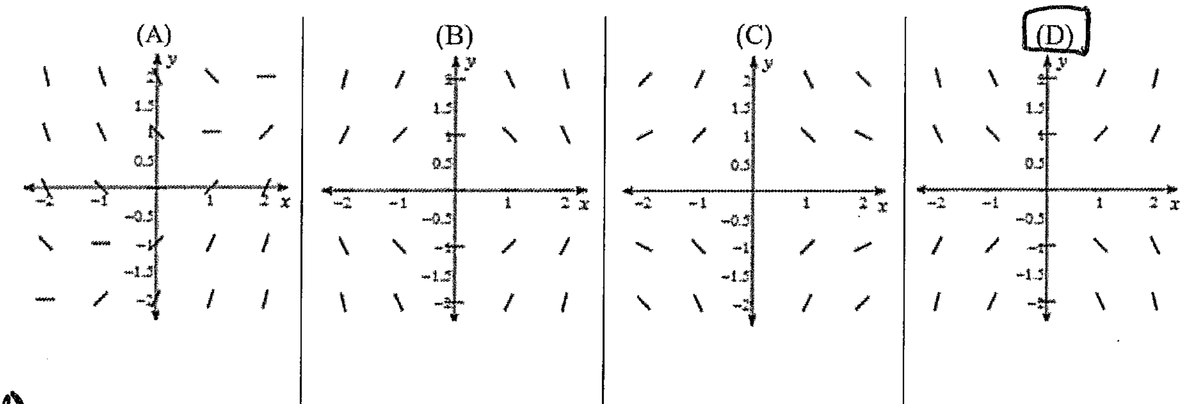
(D) $\frac{dy}{dx} = x + y$

(E) $\frac{dy}{dx} = 0.5y$



Match the slope field with the differential equation.

c). $\frac{dy}{dx} = xy$



d).

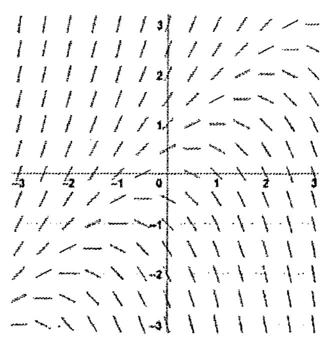
(A) $\frac{dy}{dx} = x - y$

(B) $\frac{dy}{dx} = x + y$

(C) $\frac{dy}{dx} = (x-1)(y-1)$

$\boxed{\text{(D)}}$ $\frac{dy}{dx} = y - x$

(E) $\frac{dy}{dx} = xy^2$



Solving Differential Equations Task (Continued)

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$\int \frac{-1\,du}{u}$

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$e^{\ln|100-B|} = e^{-\frac{1}{5}t + c}$

$|100-B| = e^{-1/5 t} \cdot e^{c}$

$|100-B| = e^{-\frac{1}{5}t} \cdot C$

$|100-B| = Ce^{-\frac{1}{5}t}$

$100 - B = Ce^{-\frac{1}{5}t}$

t B

(time, Bird weight)

$(0, 20)$

$100 - Ce^{-\frac{1}{5}t} = B$

$B = 100 - Ce^{-\frac{1}{5}t}$ (general equation)

plug in $(0,20)$

$20 = 100 - Ce^{-\frac{1}{5}(0)}$

$20 = 100 - C$

$C = 80$

$$\boxed{B = 100 - 80e^{-\frac{1}{5}t} \qquad B(t) = 100 - 80e^{-\frac{1}{5}t}}$$