BC Calculus Units 6-8 Quiz Review WS 1

Key

Evaluate the below: Show all work!

*Integration By Parts

1) $\int \frac{2x}{3} \ln 4x \, dx$

$f = \ln(4x) \qquad g' = \frac{2}{3}x$

$f' = \frac{4}{4x} = \frac{1}{x} \qquad g = \frac{2}{3} \cdot \frac{x^2}{2} = \frac{x^2}{3}$

IBP

$* \int fg' = fg - \int f'g$

$= \frac{x^2 \ln(4x)}{3} - \int \frac{1}{x} \cdot \frac{x^2}{3} dx$

$= \frac{x^2 \ln(4x)}{3} - \frac{1}{3}\int x \, dx$

$= \frac{x^2 \ln(4x)}{3} - \frac{1}{3} \cdot \frac{x^2}{2} + C$

$= \boxed{\frac{x^2 \ln(4x)}{3} - \frac{x^2}{6} + C}$

2) (Show your work – non- calculator)

$\int_0^6 \frac{1}{\sqrt{6-x}} \, dx$

*vertical asymptote at $x=6$

$\lim_{b \to 6^-} \int_0^b \frac{1}{(6-x)^{1/2}} dx$

$\int (6-x)^{-1/2} dx$

$u = 6-x \quad | \quad dx = -1du$

$\frac{du}{dx} = -1$

$\int u^{-1/2} \cdot -1du \to -\frac{u^{1/2}}{1/2} \to -2(6-x)^{1/2} \Big]_0^b$

$\lim_{b \to 6^-} -2(6-b)^{1/2} - (-2(6)^{1/2})$

$0 + 2\sqrt{6} = \boxed{2\sqrt{6}}$

3) $\int (3x+1) \cos 5x \, dx$

*Tab Method

	f	g'
+	$3x+1$	$\cos(5x)$
−	3	$\frac{1}{5}\sin(5x)$
+	0	$-\frac{1}{25}\cos(5x)$

$* \int \cos(5x) dx$

$u = 5x \quad | \quad dx = \frac{du}{5}$

$\frac{du}{dx} = 5$

$\int \cos u \cdot \frac{du}{5} \to \frac{1}{5}\sin u$

$\to \frac{1}{5}\sin(5x)$

$\boxed{\frac{(3x+1)}{5}\sin(5x) + \frac{3}{25}\cos(5x) + C}$

4) $\int x^3 e^{2x}\,dx$

*Tab Method

	f	g'
+	x^3	e^{2x}
−	$3x^2$	$\frac{1}{2}e^{2x}$
+	$6x$	$\frac{1}{4}e^{2x}$
−	6	$\frac{1}{8}e^{2x}$
+	0	$\frac{1}{16}e^{2x}$

$\int e^{2x}dx$ $\nearrow$ $u=2x$, $\frac{du}{dx}=2$, $dx=\frac{du}{2}$

$\int e^{u}\cdot\frac{du}{2}$

$\frac{1}{2}\int e^{u}du$

$\frac{1}{2}e^{2x}$

$$\boxed{\frac{1}{2}x^3e^{2x}-\frac{3}{4}x^2e^{2x}+\frac{3}{4}xe^{2x}-\frac{3}{8}e^{2x}+C}$$

5) $\int \frac{1}{x^2+6x+8}\,dx$

$$\frac{1}{(x+2)(x+4)}=\frac{A}{x+2}+\frac{B}{x+4}$$

$x=-2$ $\quad$ $x=-4$

$\int\frac{1}{(x+2)(x+4)}dx$ $\quad$ $A=\frac{1}{2}$ $\quad$ $B=\frac{1}{-2}$

$\int\frac{1/2}{x+2}dx+\int\frac{-1/2}{x+4}dx$

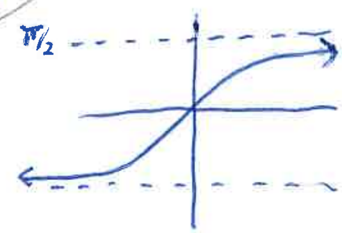
$\frac{1}{2}\int\frac{1}{x+2}dx-\frac{1}{2}\int\frac{1}{x+4}dx$

$$\boxed{\frac{1}{2}\ln|x+2|-\frac{1}{2}\ln|x+4|+C}$$

OR

$$\boxed{\frac{1}{2}\ln\left|\frac{x+2}{x+4}\right|+C}$$

$y=\arctan x$



6) $\int_0^\infty \frac{1}{9+x^2}\,dx$

$\lim_{b\to\infty}\int_0^b\frac{1}{(3)^2+(x)^2}dx$

$\lim_{b\to\infty}\left.\frac{1}{3}\arctan\left(\frac{x}{3}\right)\right]_0^b$

$\lim_{b\to\infty}\frac{1}{3}\arctan\left(\frac{b}{3}\right)-\frac{1}{3}\arctan(0)$

$\frac{1}{3}(\pi/2)-0\quad=\boxed{\pi/6}$

$*\int\frac{du}{a^2+u^2}=\frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

7)

Given that $y = f(t)$ is a solution to the logistic differential equation $\frac{dy}{dt} = \frac{y}{5} - \frac{y^2}{1500}$, where t is time in years. What is $\lim_{t\to\infty} f(t)$?

$* \ \frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$

$L = 300$

$\frac{dy}{dt} = \frac{1}{5}y\left(1 - \frac{y}{300}\right)$

$\lim_{t\to\infty} f(t) = 300$ since the Limiting value is 300.

8)

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 3x + y$ with initial condition $f(0) = 1$. What is the approximation for $f(0.5)$ obtained using Euler's method with 2 steps of equal length, starting at $x = 0$?

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{0.5-0}{2} = 0.25$

$* \ y - y_1 = m(x - x_1)$

x	y_0	$y' = 3x + y$	$y = y_0 + y'\Delta x$
0	1	$y' = 3(0) + 1 = 1$	$y = 1 + 1(0.25) = 1.25$
0.25	1.25	$y' = 3(0.25) + 1.25 = 2$	$y = 1.25 + 2(0.25) = 1.75$
0.5	1.75		

$\boxed{f(0.5) \approx 1.75}$

9)

The table below gives the values of f', the derivative of f. If $f(4.2) = 5$, what is the approximation to $f(4.6)$ obtained by using Euler's method with 2 steps of equal size?

x	4	4.2	4.4	4.6	4.8
$f'(x)$	0.2	0.3	0.5	0.61	0.73

$\Delta x = \frac{4.6 - 4.2}{2} = 0.2$

x	y_0	$y' = f'(x)$	$y = y_0 + f'(x)(\Delta x)$
4.2	5	0.3	$y = 5 + 0.3(0.2) = 5.06$
4.4	5.06	0.5	$y = 5.06 + 0.5(0.2) = 5.16$
4.6	$\boxed{5.16}$		

$\boxed{f(4.6) \approx 5.16}$

10) $\lim_{x\to 0+} x^{x^2}$

$y = x^{x^2}$

$\ln y = \ln x^{x^2}$

$\ln y = x^2 \cdot (\ln x)$

$\ln y = \dfrac{\ln x}{\frac{1}{x^2}}$

$\lim_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x^2}} \to \dfrac{-\infty}{\infty}$

L'Hopital's $\lim_{x\to 0^+} \dfrac{\ln x}{x^{-2}}$

$\lim_{x\to 0^+} \dfrac{\frac{1}{x}}{-2x^{-3}} \to \dfrac{\frac{1}{x}}{\frac{-2}{x^3}} \to \frac{1}{x} \cdot \frac{x^3}{-2}$

$\lim_{x\to 0^+} \dfrac{-x^2}{-2} \to 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y=1}$

11) $\lim_{x\to\infty} (1+x^2)^{1/x}$

$y = (1+x^2)^{1/x}$

$\ln y = \ln(1+x^2)^{1/x}$

$\ln y = \frac{1}{x}\ln(1+x^2)$

$\lim_{x\to\infty} \dfrac{\ln(1+x^2)}{x} \to \dfrac{\infty}{\infty}$

L'H $\to \lim_{x\to\infty} \dfrac{\frac{2x}{1+x^2}}{1} \to \dfrac{\infty}{\infty}$

L'H $\to \lim_{x\to\infty} \dfrac{0}{0+2x} \to 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y=1}$

12) $\lim_{x\to 0} (\cos x)^{1/x}$

$\ln y = \ln(\cos x)^{1/x}$

$\ln y = \frac{1}{x}\ln(\cos x)$

$\lim_{x\to 0} \dfrac{\ln(\cos x)}{x} \to \dfrac{0}{0}$

L'H $\to \lim_{x\to 0} \dfrac{\frac{-\sin x}{\cos x}}{1} \to \lim_{x\to 0} \dfrac{-\sin x}{\cos x} \to \dfrac{0}{1} = 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y=1}$

$f'(x) = 12x^3 + 2x - 2$

13) Find an integral that is equal to the length of the curve $f(x) = 3x^4 + x^2 - 2x + 1$ from the points $(0,1)$ to $(2, 49)$. **Do Not Evaluate.**

$$*L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$\boxed{L = \int_0^2 \sqrt{1 + [12x^3 + 2x - 2]^2}\, dx}$$

14) The length of the curve $y = 2x^3$ from $x = 1$ to $x = 5$ is given by

A. $\int_1^5 \sqrt{1 + 4x^4}\, dx$

B. $\int_1^5 \sqrt{1 + 6x^2}\, dx$

C. $\boxed{\int_1^5 \sqrt{1 + 36x^4}\, dx}$

D. $\int_1^5 \sqrt{1 + x^6}\, dx$

E. $\int_1^5 \sqrt{1 + 36x^3}\, dx$

$y' = 6x^2$

$$L = \int_1^5 \sqrt{1 + [6x^2]^2}\, dx$$

$$\boxed{L = \int_1^5 \sqrt{1 + 36x^4}\, dx}$$

Formulas to know(memorize) for quiz:

1) Integration by Parts (IBP): $\int fg' = fg - \int f'g$ Use original IBP formula if log or natural log is involved. Can only use Tab method if no logs is in the integrand

2) Improper Integrals: Remember to take limit to approach a bound that is ∞ or a point of discontinuity (vertical asymptote)

3) Partial Fraction Decomposition: Use the "cover-up" method when breaking down rational functions into separate fractions each with its own linear factor. Integral of these separate expressions will be in the form of $\int \frac{1}{u} du = \ln|u| + C$

4) Euler's Method: Create Table:

x	y_o	y' or $\frac{dy}{dx}$ or $f'(x)$	$y = y_o + y'(\Delta x)$

- $\Delta x = \frac{b-a}{n}$

5) Logistic Differential Equation: $\frac{dy}{dt} = ky(1 - \frac{y}{L})$ or $\frac{dy}{dt} = \frac{k}{L} y(L - y)$

6) Arc Length of Curve Formula: $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

7) L'Hopital's Rule: For indeterminate form in terms of $(variable)^{(variable)}$, use log differentiation to arrange in a form that L'Hopital's Rule can be applied: $\lim_{x \to a} \frac{f(x)}{g(x)} \rightarrow \lim_{x \to a} \frac{f'(x)}{g'(x)}$

BC Calculus Units 6-8 Quiz Review WS 2

Key

Evaluate the below: Show all work!

1) *Integration by Parts (IBP)

The function f has a continuous derivative. The table gives the values of f and its derivatives for $x = 2$ and $x = 7$. If $\int_2^7 f(x)\,dx = 10$, what is the value of $\int_2^7 2xf'(x)\,dx$?

x	$f(x)$	$f'(x)$
2	3	5
7	9	−4

*IBP

	f	g'
+	$2x$	$f'(x)$
−	2	$f(x)$
+	0	$\int f(x)$

$2x\cdot f(x) - 2\int f(x)\,dx$

$2x\,f(x)\Big]_2^7 - 2\underbrace{\int_2^7 f(x)\,dx}_{10}$

$2(7)\cdot f(7) - 2(2)\cdot f(2)$

$14(9) - 4(3) = 114$

$114 - 2(10)$

$114 - 20 = \boxed{94}$

2) $\int 3x \ln x^2\, dx$

IBP * $\int fg' = fg - \int f'g$

$\int 3x \cdot 2\ln x\, dx$

$\int 6x \ln x\, dx$

$f = \ln x \qquad g' = 6x\,dx$

$f' = \frac{1}{x} \qquad g = \frac{6x^2}{2} = 3x^2$

$= 3x^2 \ln x - \int \frac{1}{x}\cdot 3x^2\, dx$

$= 3x^2 \ln x - \int 3x\, dx$

$\boxed{3x^2 \ln x - \frac{3x^2}{2} + C}$

or

$\frac{3}{2}x^2 \ln x^2 - \frac{3}{2}x^2 + C$

3) $\int x \cos 4x\, dx$

*Tab Method

	f	g'
+	x	$\cos(4x)$
−	1	$\frac{1}{4}\sin(4x)$
+	0	$\frac{-1}{16}\cos(4x)$

$\int \cos(4x)\,dx$

$u = 4x$

$\frac{du}{dx} = 4$

$dx = \frac{du}{4}$

$\int \cos u \cdot \frac{du}{4}$

$\frac{1}{4}\int \cos u\, du$

$\frac{1}{4}\sin(4x)$

$\boxed{\frac{1}{4}x\sin(4x) + \frac{1}{16}\cos(4x) + C}$

4) $\int \frac{4x+1}{2x^2-3x-2}\,dx$

*Partial Fraction Decomposition

$(2x+1)(x-2)$

$$\frac{4x+1}{(2x+1)(x-2)} = \frac{A}{2x+1} + \frac{B}{x-2}$$

$x=-\frac{1}{2}$ $\quad$ $x=2$

$$\frac{4(-\frac{1}{2})+1}{(-\frac{1}{2}-2)} = \frac{-1}{-\frac{5}{2}} = \frac{2}{5} = A$$

$$\frac{4(2)+1}{2(2)+1} = \frac{9}{5} = B$$

$$\int \frac{2/5}{2x+1} + \frac{9/5}{x-2}\,dx$$

$u=2x+1$

$\frac{du}{dx}=2$

$dx=\frac{du}{2}$

$$\frac{2}{5}\int \frac{1}{u}\cdot\frac{du}{2}$$

$$\boxed{\frac{1}{5}\ln|2x+1| + \frac{9}{5}\ln|x-2| + C}$$

5) $\int_2^\infty \frac{3}{x(x-1)}\,dx$

VA: $x=0, x=1$ (no impact)

*Partial Fraction first

$$\frac{3}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$x=0$ $\quad$ $x=1$

$$\frac{3}{0-1} = -3 = A \qquad \frac{3}{1} = B$$

$$\lim_{b\to\infty}\int_2^b \frac{3}{x} + \frac{3}{x-1}\,dx$$

$$3\ln|x| + 3\ln|x-1|\Big]_2^b$$

$$\lim_{b\to\infty} 3\ln|b| + 3\ln|b-1| - (3\ln 2 + 3\ln 1) = \boxed{\text{Diverges}}$$

6) Non-Calculator (Show your work)

$$\int_0^5 \frac{1}{\sqrt{5-x}}\,dx$$

$5-x=0$

$x=5$

VA at $x=5$ (Discontinuity at $x=5$)

$$\lim_{b\to 5^-}\int_0^b \frac{1}{(5-x)^{1/2}}\,dx$$

$u=5-x$

$\frac{du}{dx}=-1$

$dx=-1du$

$$\int (u)^{-1/2}(-1\,du)$$

$$\frac{-u^{1/2}}{1/2} \to -2u^{1/2}$$

$$-2(5-x)^{1/2}\Big]_0^b$$

$$\lim_{b\to 5^-} -2(5-b)^{1/2} - \left(-2(5-0)^{1/2}\right)$$

$$-2(0)^{1/2} + 2\sqrt{5}$$

$$= \boxed{2\sqrt{5}}$$

7)

$n=2$

Let $h(x) = \int_0^x \sqrt{1+4t^2}\,dt$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(3)$.

$\Delta x = \frac{b-a}{n} \rightarrow \frac{3-0}{2} = 1.5$

$h'(x) = \frac{d}{dx}\int_0^x \sqrt{1+4t^2}\,dt$

$h'(x) = \sqrt{1+4x^2}$

x	y_0	$h'(x) = \sqrt{1+4x^2}$	$y = y_0 + h'(x)(\Delta x)$
0	0	$h'(0) = \sqrt{1+0} = 1$	$y = 0 + 1(1.5) = 1.5$
1.5	1.5	$h'(1.5) = \sqrt{1+4(1.5)^2} = 3.1622$	$y = 1.5 + 3.1622(1.5) = 6.243$
3	$\boxed{6.243}$		

$\boxed{h(3) \approx 6.243}$

8)

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with initial condition $f(0) = 3$. What is the approximation for $f(0.5)$ obtained using Euler's method with 2 steps of equal length, starting at $x = 0$?

$\Delta x = \frac{b-a}{n} = \frac{0.5-0}{2}$

$\Delta x = \frac{1}{4} = 0.25$

x	y_0	$\frac{dy}{dx} = x+y$	$y = y_0 + y'(\Delta x)$
0	3	$y' = 0+3 = 3$	$y = 3 + 3(0.25) = 3.75$
0.25	3.75	$y' = 0.25+3.75 = 4$	$y = 3.75 + 4(0.25) = 4.75$
0.5	$\boxed{4.75}$		

$\boxed{f(0.5) \approx 4.75}$

9)

A populations rate of growth is modeled by the logistic differential equation $\frac{dP}{dt} = \frac{1}{1000}P(600-P)$, where t is in days and $P(0) = 60$. What is the greatest rate of change for this population? * set $P''(t) = 0$

$P' = 0.6P - \frac{1}{1000}P^2$

$P' = 0.6P - 0.001P^2$

$P'' = 0.6 - 0.002P$

$0 = 0.6 - 0.002P$

$0.002P = 0.6$

$P = 300$

$\left.\frac{dP}{dt}\right|_{P=300} = \frac{1}{1000}(300)[600-300]$

$= \boxed{90 \text{ people/day}}$

10)

$L = \_\_?$

Using the logistic differential equation $\frac{dP}{dt} = \frac{1}{5}P - \frac{1}{2000}P^2$, identify the carrying capacity.

$* \frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$

$\frac{dP}{dt} = \frac{1}{5}P\left[1 - \frac{P}{400}\right]$

Carrying capacity is 400 since $L = 400$

11)

$L = 1000$

A rate of change $\frac{dP}{dt}$ of a population is modeled by a logistic differential equation. If $\lim_{t\to\infty} P(t) = 1000$ and the rate of change of the population is 100 when the population size is 50, which of the following differential equations describe the situation?

A. $\frac{dP}{dt} = 50P\left(1 - \frac{P}{1000}\right)$

B. $\frac{dP}{dt} = 100P\left(1 - \frac{P}{1000}\right)$

C. $\frac{dP}{dt} = \frac{19}{40}P\left(1 - \frac{P}{1000}\right)$

D. $\boxed{\frac{dP}{dt} = \frac{40}{19}P\left(1 - \frac{P}{1000}\right)}$

$* \frac{dP}{dt} = 100$ when $P = 50$

* test the answer choices to find solution

$\frac{dP}{dt} = \frac{40}{19}P\left[1 - \frac{P}{1000}\right]$

$100 = \frac{40}{19}(50)\left[1 - \frac{50}{1000}\right]$

$100 = 100$ ✓

12)

No Calculator. Suppose $F(x) = \int_0^x \sqrt{3 - 4\sin^2 t}\, dt$. What is the length of the arc along the curve $y = F(x)$ for $0 \le x \le \frac{\pi}{3}$?

$* L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$f'(x) = \frac{d}{dx}\int_0^x \sqrt{3 - 4\sin^2 t} = \sqrt{3 - 4\sin^2 x}$

$L = \int_0^{\pi/3} \sqrt{1 + \left[\sqrt{3 - 4\sin^2 x}\right]^2}\, dx = \int_0^{\pi/3} \sqrt{1 + 3 - 4\sin^2 x}$

$\int_0^{\pi/3} \sqrt{4 - 4\sin^2 x}$

$* 4\sin^2 x + 4\cos^2 x = 4$

$4 - 4\sin^2 x = 4\cos^2 x$

$\int_0^{\pi/3} \sqrt{4\cos^2 x}\, dx = \int_0^{\pi/3} 2\cos x\, dx$

$\int_0^{\pi/3} 2\cos x\, dx \to 2\sin x\Big]_0^{\pi/3}$

$2\sin(\pi/3) - 2\sin(0)$

$= 2 \cdot \frac{\sqrt{3}}{2} - 0$

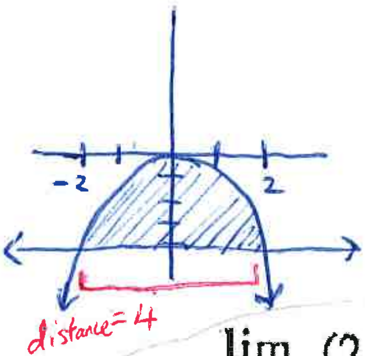
$L = \boxed{\sqrt{3}}$

$\boxed{L = \sqrt{3}}$

13)

Let R be the region bounded by the graphs of $f(x) = -x^2$ and $g(x) = -4$. Write an expression including one or more integrals that gives the perimeter of the region R. **Do Not Evaluate.**

$$*L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$



Perimeter of graph $= 4 + \int_{-2}^{2} \sqrt{1+[-2x]^2}\,dx$ $\qquad f'(x) = -2x$

$$\to \boxed{4 + \int_{-2}^{2} \sqrt{1+4x^2}\,dx}$$

14) $\lim_{x\to 0+} (2x)^{3x}$

$y = (2x)^{3x}$

$\ln y = \ln(2x)^{3x}$

$\ln y = 3x \cdot \ln(2x)$

$\lim_{x\to 0^+} \dfrac{\ln(2x)}{\frac{1}{3x}} \to \dfrac{-\infty}{\infty}$ ← Indeterminate Form (Apply L'Hopital's Rule)

L'H → $\lim_{x\to 0^+} \dfrac{\ln(2x)}{(3x)^{-1}} \to \dfrac{\frac{2}{2x}}{-1(3x)^{-2}(3)} \to \dfrac{\frac{1}{x}}{\frac{-3}{(3x)^2}} \to \dfrac{\frac{1}{x}}{\frac{-1}{3x^2}} \to \dfrac{1}{x} \cdot \dfrac{-3x^2}{1}$

$\lim_{x\to 0^+} \dfrac{-3x}{1} \to 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y = 1}$

15) $\lim_{x\to 0+} (\csc x)^{\sin x}$

$y = (\csc x)^{\sin x}$

$\ln y = \ln(\csc x)^{\sin x}$

$\ln y = \sin x \cdot \ln(\csc x)$

$\ln y = \dfrac{\ln(\csc x)}{\csc x}$

$\lim_{x\to 0^+} \dfrac{\ln(\csc x)}{\csc x} \to \dfrac{-\infty}{\infty}$

$\lim_{x\to 0^+} \dfrac{\frac{-\csc x \cot x}{\csc x}}{-\csc x \cot x} \to \dfrac{-\cancel{\csc x \cot x}}{\csc x} \cdot \dfrac{1}{-\cancel{\csc x \cot x}}$

$\lim_{x\to 0^+} \dfrac{1}{\csc x} \to \lim_{x\to 0^+} \sin x = 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y = 1}$

BC Calculus Units 6-8 Quiz Review WS 3

Key

Evaluate the below: Show all work!

1) Integration by Parts (IBP)

The function f has a continuous derivative. The table gives the values of f and its derivatives for $x = 1$ and $x = 6$. If $\int_1^6 f(x)\,dx = 9$, what is the value of $\int_1^6 3xf'(x)\,dx$?

*IBP (Tab Method)

x	$f(x)$	$f'(x)$
1	2	4
6	8	-3

	f	g'
+	$3x$	$f'(x)$
−	3	$f(x)$
+	0	$\int f(x)$

$$\int_1^6 3xf'(x)dx = 3xf(x) - \int 3f(x)dx \Big]_1^6$$

$$3xf(x)\Big]_1^6 - 3\int_1^6 f(x)dx \to 9$$

$$3(6)\cdot f(6) - 3(1)\cdot f(1)$$

$$= 18(8) - 3(2) - 3(9) = 144 - 6 - 27 = \boxed{111}$$

2) $\int 4xe^{3x+2}dx$

*IBP (Tab Method)

	f	g'
+	$4x$	e^{3x+2}
−	4	$\frac{1}{3}e^{3x+2}$
+	0	$\frac{1}{9}e^{3x+2}$

$\int e^{3x+2}dx$ | $u = 3x+2$, $\frac{du}{dx} = 3$, $dx = \frac{du}{3}$ | $\int e^u \cdot \frac{du}{3}$, $\frac{1}{3}\int e^u du$ | $\frac{1}{3}e^u \to \frac{1}{3}e^{3x+2}$

$$\boxed{\frac{4}{3}xe^{3x+2} - \frac{4}{9}e^{3x+2} + C}$$

3) $\int \frac{2}{(x-1)(x+2)(x-4)}dx$

$$\frac{2}{(x-1)(x+2)(x-4)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-4}$$

$x=1$ $\quad x=-2$ $\quad x=4$

$$\frac{2}{(1+2)(1-4)} \to \frac{2}{3(-3)} = \frac{2}{-9} = A$$

$$\frac{2}{(-2-1)(-2-4)} \to \frac{2}{(-3)(-6)} \to \frac{1}{9} = B$$

$$\frac{2}{(4-1)(4+2)} \to \frac{2}{3(6)} \to \frac{2}{18} = \frac{1}{9} = C$$

$$\int \frac{-2/9}{x-1} + \frac{1/9}{x+2} + \frac{1/9}{x-4}dx$$

$$\boxed{-\frac{2}{9}\ln|x-1| + \frac{1}{9}\ln|x+2| + \frac{1}{9}\ln|x-4| + C}$$

4) $\int_2^{\infty} x^{-3}\ dx$

$\lim_{b\to\infty} \int_2^b x^{-3} dx \qquad \frac{x^{-2}}{-2} \to -\frac{1}{2x^2}\Big]_2^b$

$\lim_{b\to\infty} \frac{-1}{2b^2} - \left(\frac{-1}{2(2)^2}\right)$

$0 + \frac{1}{8} = \boxed{\frac{1}{8}}$

VA at $x=3$ (Discontinuity at $x=3$)

5) $\int_0^3 \frac{1}{\sqrt{9-x^2}}\ dx$

$9-x^2=0$
$x=3$

$\lim_{b\to 3^-} \int_0^b \frac{1}{\sqrt{a^2-u^2}} dx$

$\arcsin\left(\frac{x}{3}\right)\Big]_0^b = \arcsin\left(\frac{b}{3}\right) - \arcsin\left(\frac{0}{3}\right)$

* $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)$

$\lim_{b\to 3^-} \arcsin\left(\frac{b}{3}\right) - \arcsin 0$

$\int \frac{1}{\sqrt{3^2-x^2}} \to \arcsin\left(\frac{x}{3}\right)$

$= \arcsin(1) - 0$

$= \boxed{\frac{\pi}{2}}$

6)

Let $h(x) = \int_1^x \frac{1}{t^2} dt$. Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $h(3)$.

$\Delta x = \frac{b-a}{n} \to \frac{3-1}{2} = \frac{2}{2}$

$\Delta x = 1$

$h'(x) = \frac{d}{dx}\int_1^x \frac{1}{t^2} dt$

$h'(x) = \frac{1}{x^2}$

x	y_0	$h'(x) = \frac{1}{x^2}$	$y = y_0 + h'(x)(\Delta x)$
1	$h(1) = \int_1^1 \frac{1}{t^2} = 0$	$h'(1) = \frac{1}{1^2} = 1$	$y = 0 + 1(1) = 1$
2	1	$h'(2) = \frac{1}{2^2} = \frac{1}{4} = 0.25$	$y = 1 + 0.25(1) = 1.25$
3	$\boxed{1.25}$		

$\boxed{h(3) \approx 1.25}$

7)

The table below gives the values of f', the derivative of f. If $f(2) = 1$, what is the approximation to $f(2.3)$ obtained by using Euler's method with 3 steps of equal size?

x	2	2.1	2.2	2.3
$f'(x)$	-0.1	-0.15	-0.3	-0.5

$\Delta x = \frac{b-a}{n} \rightarrow \frac{2.3-2}{3}$

$\Delta x = 0.1$

x	y_0	$f'(x)$	$y = y_0 + f'(x)(\Delta x)$
2	1	$f'(2) = -0.1$	$y = 1 + (-0.1)(0.1) = 0.99$
2.1	0.99	$f'(2.1) = -0.15$	$y = 0.99 + (-0.15)(0.1) = 0.975$
2.2	0.975	$f'(2.2) = -0.3$	$y = 0.975 + (-0.3)(0.1) = 0.945$
2.3	$\boxed{0.945}$		

$\boxed{f(2.3) \approx 0.945}$

8)

A populations rate of growth is modeled by the logistic differential equation $\frac{dP}{dt} = \frac{1}{400}P(100 - P)$, where t is in days and $P(0) = 10$. What is the greatest rate of change for this population? * set $P'' = 0$

$\frac{dP}{dt} = \frac{100}{400}P - \frac{1}{400}P^2$

$P' = 0.25P - 0.0025P^2$

$P'' = 0.25 - 0.005P$

$0 = 0.25 - 0.005P$

$0.005P = 0.25$

$P = 50$

$\left.\frac{dP}{dt}\right|_{P=50} = \frac{1}{400}(50)[100-50]$

$= \boxed{\frac{25}{4} \text{ people per day}}$

9)

Using the logistic differential equation $\frac{dP}{dt} = \frac{1}{3}P - \frac{1}{120}P^2$, what is $\lim_{t\to\infty} P(t)$?

* $\frac{dy}{dt} = ky\left[1 - \frac{y}{L}\right]$

$\frac{dP}{dt} = \frac{1}{3}P\left[1 - \frac{P}{40}\right]$

* Limiting value is 40

$\boxed{\lim_{t\to\infty} P(t) = 40}$

10)

$L = 100$

A rate of change $\frac{dP}{dt}$ of a population is modeled by a logistic differential equation. $\boxed{\text{If } \lim_{t\to\infty} P(t) = 100}$ and the rate of change of the population is 5 when the population size is 20, which of the following differential equations describe the situation?

$*\frac{dP}{dt} = 5$, $P = 20$

A. $\frac{dP}{dt} = 5P\left(1 - \frac{P}{20}\right)$

B. $\frac{dP}{dt} = 20P\left(1 - \frac{P}{100}\right)$

$\boxed{\text{C. } \frac{dP}{dt} = \frac{5}{16}P\left(1 - \frac{P}{100}\right)}$ $\rightarrow$ $5 = \frac{5}{16}(20)\left[1 - \frac{20}{100}\right]$

$5 = 5$ ✓

D. $\frac{dP}{dt} = \frac{16}{5}P\left(1 - \frac{P}{100}\right)$

11)

A rate of change for a population is modeled by the differential equation $\frac{dP}{dt} = 0.3P(66 - P)$. What is the population when the rate of change is the greatest? $*$ set $P'' = 0$

$P' = 19.8P - 0.3P^2$

$P'' = 19.8 - 0.6P$

$0 = 19.8 - 0.6P$

$0.6P = 19.8$

$P = 33$

$\boxed{\text{Population when rate of change is greatest is 33}}$

12) $* \; L = \int_a^b \sqrt{1 + [F'(x)]^2}\,dx$

No Calculator. Suppose $F(x) = \int_0^x \sqrt{3 - 4\cos^2 t}\; dt$. What is the length of the arc along the curve $y = F(x)$ for $0 \le x \le \frac{\pi}{3}$?

$f'(x) = \frac{d}{dx}\int_0^x \sqrt{3 - 4\cos^2 t}\,dt$

$f'(x) = \sqrt{3 - 4\cos^2 x}$

$L = \int_0^{\pi/3} \sqrt{1 + \left[\sqrt{3 - 4\cos^2 x}\right]^2}\,dx$

$L = \int_0^{\pi/3} \sqrt{1 + 3 - 4\cos^2 x}\,dx$

$* \; 4\sin^2 x + 4\cos^2 x = 4$

$L = \int_0^{\pi/3} \sqrt{4 - 4\cos^2 x}\,dx \leftarrow 4\sin^2 x = 4 - 4\cos^2 x$

$L = \int_0^{\pi/3} \sqrt{4\sin^2 x}\,dx$

$L = \int_0^{\pi/3} 2\sin x\,dx$

$-2\cos x \Big]_0^{\pi/3}$

$-2\cos(\pi/3) - (-2\cos(0))$

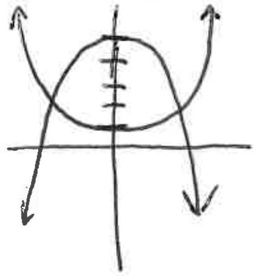
$-2(1/2) + 2$

$-1 + 2 = \boxed{1}$

13) $x^2+1=-x^2+5$ | $x^2=2$

$2x^2=4$ | $x=\pm\sqrt{2}$ $\quad f'(x)=2x \quad g'(x)=-2x$

Let R be the region bounded by the graphs of $f(x) = x^2 + 1$ and $g(x) = -x^2 + 5$. Write an expression including one or more integrals that gives the length of the region R. **Do Not Evaluate.**



$$L_1=\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{1+[f'(x)]^2}\,dx \rightarrow \int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{1+[2x]^2}\,dx \rightarrow \int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{1+4x^2}\,dx$$

$$L_2=\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{1+[g'(x)]^2}\,dx \rightarrow \int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{1+[2x]^2}\,dx \rightarrow \int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{1+4x^2}\,dx$$

$$\boxed{L_1+L_2 = 2\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{1+4x^2}\,dx}$$

14) $\lim_{x\to\infty}(x+1)^{e^{-x}}$

$y=(x+1)^{e^{-x}}$

$\ln y=\ln(x+1)^{e^{-x}}$

$\ln y=e^{-x}\cdot\ln(x+1)$

$\ln y=\dfrac{\ln(x+1)}{e^x}$

$\lim_{x\to\infty}\dfrac{\ln(x+1)}{e^x}\rightarrow\dfrac{\infty}{\infty}$

$L'H \rightarrow \lim_{x\to\infty}\dfrac{\frac{1}{x+1}}{e^x}$

$\lim_{x\to\infty}\dfrac{1}{e^x(x+1)}\rightarrow 0$

$\ln y=0$

$e^{\ln y}=e^0$

$y=e^0$

$\boxed{y=1}$

15) $\lim_{x\to\pi/2^-}(\sin x)^{\tan x}$

$y=\sin x^{\tan x}$

$\ln y=\ln(\sin x)^{\tan x}$

$\ln y=\tan x\cdot\ln(\sin x)$

$\ln y=\dfrac{\ln(\sin x)}{\cot x}$

$\lim_{x\to\pi/2^-}\dfrac{\ln(\sin x)}{\cot x}\rightarrow L'H\rightarrow\lim_{x\to\pi/2^-}\dfrac{\frac{\cos x}{\sin x}}{-\csc^2 x}$

$\dfrac{\cos x}{\sin x}\cdot\dfrac{1}{-\csc^2 x}\rightarrow\dfrac{\cos x}{\sin x}\cdot\dfrac{-\sin^2 x}{1}\rightarrow -\cos x\sin x$

$\lim_{x\to\pi/2^-}-\cos x\sin x = -\cos(\pi/2)\sin(\pi/2)\rightarrow -0(1)=0$

$\ln y=0$

$e^{\ln y}=e^0$

$y=e^0$

$\boxed{y=1}$