Key

BC Calculus Units 6-8 Quiz Review WS 1

Evaluate the below: Show all work!

*Integration By Parts

1) $\int \frac{2x}{3} \ln 4x \, dx$

$f = \ln(4x)$ $\quad g' = \frac{2}{3}x$

$f' = \frac{4}{4x} = \frac{1}{x}$ $\quad g = \frac{2}{3} \cdot \frac{x^2}{2} = \frac{x^2}{3}$

IBP

$* \int fg' = fg - \int f'g$

$= \frac{x^2 \ln(4x)}{3} - \int \frac{1}{x} \cdot \frac{x^2}{3} dx$

$= \frac{x^2 \ln(4x)}{3} - \frac{1}{3}\int x \, dx$

$= \frac{x^2 \ln(4x)}{3} - \frac{1}{3} \cdot \frac{x^2}{2} + C$

$= \boxed{\frac{x^2 \ln(4x)}{3} - \frac{x^2}{6} + C}$

2) (Show your work – non- calculator)

$\int_0^6 \frac{1}{\sqrt{6-x}} \, dx$

*vertical asymptote at $x=6$

$\lim_{b \to 6^-} \int_0^b \frac{1}{(6-x)^{1/2}} dx$

$\int (6-x)^{-1/2} dx$

$u = 6-x$ $\quad dx = -1du$

$\frac{du}{dx} = -1$

$\int u^{-1/2} \cdot -1du \to \frac{-u^{1/2}}{1/2} \to -2(6-x)^{1/2} \Big]_0^b$

$\lim_{b \to 6^-} -2(6-b)^{1/2} - (-2(6)^{1/2})$

$0 + 2\sqrt{6} = \boxed{2\sqrt{6}}$

3) $\int (3x+1) \cos 5x \; dx$

*Tab Method

	f	g'
+	$3x+1$	$\cos(5x)$
−	3	$\frac{1}{5}\sin(5x)$
+	0	$-\frac{1}{25}\cos(5x)$

$* \int \cos(5x) dx$

$u = 5x$ $\quad dx = \frac{du}{5}$

$\frac{du}{dx} = 5$

$\int \cos u \cdot \frac{du}{5} \to \frac{1}{5}\sin u$

$\to \frac{1}{5}\sin(5x)$

$\boxed{\frac{(3x+1)}{5}\sin(5x) + \frac{3}{25}\cos(5x) + C}$

4) $\int x^3 e^{2x}\, dx$

*Tab Method

	f	g'
+	x^3	e^{2x}
−	$3x^2$	$\frac{1}{2}e^{2x}$
+	$6x$	$\frac{1}{4}e^{2x}$
−	6	$\frac{1}{8}e^{2x}$
+	0	$\frac{1}{16}e^{2x}$

$\int e^{2x} dx$ $\quad u = 2x$, $\frac{du}{dx} = 2$, $dx = \frac{du}{2}$

$\int e^{u} \cdot \frac{du}{2}$

$\frac{1}{2}\int e^{u} du$

$\frac{1}{2}e^{2x}$

$$\boxed{\frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + C}$$

5) $\int \frac{1}{x^2+6x+8}\, dx$

$\frac{1}{(x+2)(x+4)} = \frac{A}{x+2} + \frac{B}{x+4}$

$x=-2$ $\quad x=-4$

$\int \frac{1}{(x+2)(x+4)} dx$

$A = \frac{1}{2}$ $\quad B = \frac{1}{-2}$

$\int \frac{1/2}{x+2} dx + \int \frac{-1/2}{x+4} dx$

$\frac{1}{2}\int \frac{1}{x+2} dx - \frac{1}{2}\int \frac{1}{x+4} dx$

$$\boxed{\frac{1}{2}\ln|x+2| - \frac{1}{2}\ln|x+4| + C}$$

OR

$$\boxed{\frac{1}{2}\ln\left|\frac{x+2}{x+4}\right| + C}$$

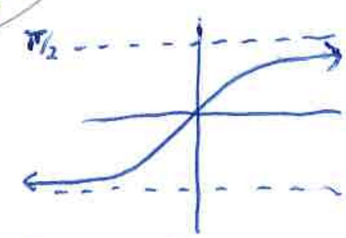
6) $\int_0^{\infty} \frac{1}{9+x^2}\, dx$

$\lim_{b\to\infty} \int_0^b \frac{1}{(3)^2+(x)^2} dx$

$\lim_{b\to\infty} \frac{1}{3}\arctan\left(\frac{x}{3}\right)\Big]_0^b$

$\lim_{b\to\infty} \frac{1}{3}\arctan\left(\frac{b}{3}\right) - \frac{1}{3}\arctan(0)$ ← $\tan\theta = 0$ so $\theta = 0$

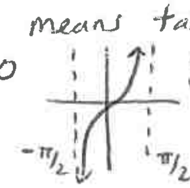
$\frac{1}{3}\left(\pi/2\right) - 0 \quad = \boxed{\pi/6}$



* $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$

OR: $\arctan(\infty)$ means $\tan\theta = \infty$. What θ will make $\tan\theta = \infty$

$\theta = \pi/2$



7)

Given that $y = f(t)$ is a solution to the logistic differential equation $\frac{dy}{dt} = \frac{y}{5} - \frac{y^2}{1500}$, where t is time in years. What is $\lim_{t\to\infty} f(t)$?

factor $\frac{y}{5}$ out from expression

$$* \frac{dy}{dt} = ky\left(1-\frac{y}{L}\right)$$

$L = 300$

$$\frac{y}{5}\left(1-\frac{y}{300}\right)$$

$$\frac{dy}{dt} = \frac{1}{5}y\left(1-\frac{y}{300}\right)$$

$\lim_{t\to\infty} f(t) = 300$ since the Limiting value is 300.

8)

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 3x + y$ with initial condition $f(0) = 1$. What is the approximation for $f(0.5)$ obtained using Euler's method with 2 steps of equal length, starting at $x = 0$?

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{0.5-0}{2} = 0.25$$

$*y - y_1 = m(x - x_1)$

x	y_0	$y' = 3x + y$	$y = y_0 + y'\Delta x$
0	1	$y' = 3(0) + 1 = 1$	$y = 1 + 1(0.25) = 1.25$
0.25	1.25	$y' = 3(0.25) + 1.25 = 2$	$y = 1.25 + 2(0.25) = 1.75$
0.5	1.75		

$$\boxed{f(0.5) \approx 1.75}$$

9)

The table below gives the values of f', the derivative of f. If $f(4.2) = 5$, what is the approximation to $f(4.6)$ obtained by using Euler's method with 2 steps of equal size?

x	4	4.2	4.4	4.6	4.8
$f'(x)$	0.2	0.3	0.5	0.61	0.73

$$\Delta x = \frac{4.6-4.2}{2} = 0.2$$

x	y_0	$y' = f'(x)$	$y = y_0 + f'(x)(\Delta x)$
4.2	5	0.3	$y = 5 + 0.3(0.2) = 5.06$
4.4	5.06	0.5	$y = 5.06 + 0.5(0.2) = 5.16$
4.6	$\boxed{5.16}$		

$$\boxed{f(4.6) \approx 5.16}$$

10) $\lim_{x \to 0^+} x^{x^2}$

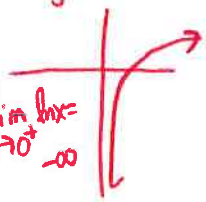
$y = x^{x^2}$

$\ln y = \ln x^{x^2}$

$\ln y = x^2 \cdot (\ln x)$

$\ln y = \dfrac{\ln x}{\frac{1}{x^2}}$

$y = \ln x$



$\lim_{x \to 0^+} \ln x = -\infty$

$\lim_{x \to 0^+} \dfrac{\ln x}{\frac{1}{x^2}} \to \dfrac{-\infty}{\infty}$

L'Hopital's $\lim_{x \to 0^+} \dfrac{\ln x}{x^{-2}}$

$\lim_{x \to 0^+} \dfrac{\frac{1}{x}}{-2x^{-3}} \to \dfrac{\frac{1}{x}}{\frac{-2}{x^3}} \to \frac{1}{x} \cdot \frac{x^3}{-2}$

$\lim_{x \to 0^+} \dfrac{-x^2}{-2} \to 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y = 1}$

11) $\lim_{x \to \infty} (1 + x^2)^{1/x}$

$y = (1+x^2)^{1/x}$

$\ln y = \ln(1+x^2)^{1/x}$

$\ln y = \frac{1}{x} \ln(1+x^2)$

$\lim_{x \to \infty} \dfrac{\ln(1+x^2)}{x} \to \dfrac{\infty}{\infty}$

$L'H \to \lim_{x \to \infty} \dfrac{\frac{2x}{1+x^2}}{1} \to \lim_{x \to \infty} \dfrac{2x}{1+x^2} \to \dfrac{2}{2x}$

$L'H \to \lim_{x \to \infty} \dfrac{2}{2x} \to 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y = 1}$

12) $\lim_{x \to 0} (\cos x)^{1/x}$

$y = (\cos x)^{1/x}$

$\ln y = \ln(\cos x)^{1/x}$

$\ln y = \frac{1}{x} \ln(\cos x)$

$\lim_{x \to 0} \dfrac{\ln(\cos x)}{x} \to \dfrac{0}{0}$

$L'H \to \lim_{x \to 0} \dfrac{\frac{-\sin x}{\cos x}}{1} \to \lim_{x \to 0} \dfrac{-\sin x}{\cos x} \to \dfrac{0}{1} = 0$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y = 1}$

$f'(x) = 12x^3 + 2x - 2$

13) Find an integral that is equal to the length of the curve $f(x) = 3x^4 + x^2 - 2x + 1$ from the points $(0,1)$ to $(2, 49)$. **Do Not Evaluate.**

$$*L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$\boxed{L = \int_0^2 \sqrt{1 + [12x^3 + 2x - 2]^2}\, dx}$$

14) The length of the curve $y = 2x^3$ from $x = 1$ to $x = 5$ is given by

A. $\int_1^5 \sqrt{1 + 4x^4}\, dx$

B. $\int_1^5 \sqrt{1 + 6x^2}\, dx$

C. $\boxed{\int_1^5 \sqrt{1 + 36x^4}\, dx}$

D. $\int_1^5 \sqrt{1 + x^6}\, dx$

E. $\int_1^5 \sqrt{1 + 36x^3}\, dx$

$y' = 6x^2$

$$L = \int_1^5 \sqrt{1 + [6x^2]^2}\, dx$$

$$\boxed{L = \int_1^5 \sqrt{1 + 36x^4}\, dx}$$

<u>Formulas to know(memorize) for quiz:</u>

1) <u>Integration by Parts (IBP):</u> $\int fg' = fg - \int f'g$ Use original IBP formula if log or natural log is involved. Can only use Tab method if no logs is in the integrand

2) <u>Improper Integrals</u>: Remember to take limit to approach a bound that is ∞ or a point of discontinuity (vertical asymptote)

3) <u>Partial Fraction Decomposition:</u> Use the "cover-up" method when breaking down rational functions into separate fractions each with its own linear factor. Integral of these separate expressions will be in the form of $\int \frac{1}{u} du = \ln|u| + C$

4) <u>Euler's Method</u>: Create Table:

x	y_o	y' or $\frac{dy}{dx}$ or $f'(x)$	$y = y_o + y'(\Delta x)$

- $\Delta x = \frac{b-a}{n}$

5) <u>Logistic Differential Equation</u>: $\frac{dy}{dt} = ky(1 - \frac{y}{L})$ or $\frac{dy}{dt} = \frac{k}{L} y(L - y)$

6) <u>Arc Length of Curve Formula:</u> $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

7) <u>L'Hopital's Rule:</u> For indeterminate form in terms of $(variable)^{(variable)}$, use log differentiation to arrange in a form that L'Hopital's Rule can be applied: $\lim_{x \to a} \frac{f(x)}{g(x)} \rightarrow \lim_{x \to a} \frac{f'(x)}{g'(x)}$