10.2-10.5 Review WS #1

(1)

A particle moves in the xy −plane in such a way that its velocity vector is $\langle 1+t, t^3 \rangle$. If the position vector at $t=0$ is $\langle 5,0 \rangle$, find the position of the particle at $t=2$.

(2)

The position of a particle at any time $t \geq 0$ is given by $x(t)=t^2-3$ and $y(t)=\frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at time $t=5$.

(b) Find the total distance traveled by the particle from $t=0$ to $t=5$.

(c) Find $\frac{dy}{dx}$ as a function of x.

(3)

Sketch curve represented by the parametric equation and write the corresponding rectangular equation by eliminating the parameter.

x= $\sqrt{t}$ y = t – 2

(4)

(Calculator) An object moving along a curve in the xy–plane has position $(x(t), y(t))$ with $\frac{dx}{dt} = \cos(t^2)$ and $\frac{dy}{dt} = \sin(t^3)$. At time $t = 0$, the object is at position $(4, 7)$. Where is the particle when $t = 2$?

(A) $\langle -0.564, 0.989\rangle$ (B) $\langle 0.461, 0.452\rangle$ (C) $\langle 3.346, 7.989\rangle$ (D) $\langle 4.461, 7.452\rangle$ (E) $\langle 5.962, 8.962\rangle$

(5)

Find the arc length of the curve on the given interval

x= $\sqrt{t}$ y = 3t – 1 $0 \le t \le 1$

(6) Find the surface area generated by revolving the curve about the given axis:

$x = 2t,\ y = 4 - 2t$ x-axis $0 \leq t \leq 2$

(7)

Find all points of horizontal and vertical tangency for $r = 1 - sin\theta$

(8) Find area of region inside $r = 3\cos\theta$ and outside $r = 2 - \cos\theta$

10.2-10.5 Review WS #1

Key

(1)

$\vec{v}(t) = \langle 1+t, t^3 \rangle$

A particle moves in the xy-plane in such a way that its velocity vector is $\langle 1+t, t^3 \rangle$. If the position vector at $t=0$ is $\langle 5,0 \rangle$, find the position of the particle at $t=2$. $\vec{s}(0) = \langle 5, 0 \rangle$

final position = initial position + displacement

$$x(2) = x(0) + \int_0^2 1+t\,dt$$
$$= 5 + 4 = 9$$
$$y(2) = y(0) + \int_0^2 t^3\,dt$$
$$= 0 + 4 = 4$$

$\vec{s}(2) = \langle 9, 4 \rangle$

or particle is at (9,4) at $t=2$.

(2)

The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at time $t=5$. $x'(t) = 2t$ $y'(t) = 2t^2$

$$\|v\| = \sqrt{x'(t)^2 + y'(t)^2}$$
$$\|\vec{v}(5)\| = \sqrt{(2t)^2 + (2t^2)^2}$$
$$= \sqrt{[2(5)]^2 + [2(5)^2]^2} = \sqrt{10^2 + 50^2} = \sqrt{2600} = \boxed{10\sqrt{26}}$$

(b) Find the total distance traveled by the particle from $t=0$ to $t=5$.

Total Distance = Arc Length $= \int_a^b \|v(t)\|\,dt$

$$\int_0^5 \sqrt{(2t)^2 + (2t^2)^2}\,dt =$$
$$\int_0^5 \sqrt{4t^2 + 4t^4}\,dt = \sqrt{4t^2(1+t^2)}\,dt$$

$$\int 2t\sqrt{1+t^2}\,dt$$
$$= \boxed{\frac{2}{3}\left[26^{3/2} - 1\right]}$$

(c) Find $\frac{dy}{dx}$ as a function of x.

* eliminate parameter first.

$$y = \frac{2}{3}t^3$$
$$\frac{3}{2}y = t^3$$
$$\left(\frac{3}{2}y\right)^{1/3} = t$$

$$x = \left[\left(\frac{3}{2}y\right)^{1/3}\right]^2 - 3$$
$$x = \left(\frac{3}{2}\right)^{2/3} y^{2/3} - 3$$
$$x + 3 = \left(\frac{3}{2}\right)^{2/3} y^{2/3}$$
$$\left(\frac{2}{3}\right)^{2/3}(x+3) = y^{2/3}$$

$$\frac{2}{3}(x+3)^{3/2} = y$$
$$\frac{dy}{dx} = \frac{2}{3}\cdot\frac{3}{2}(x+3)^{1/2}$$
$$\boxed{\frac{dy}{dx} = (x+3)^{1/2}}$$

(3)

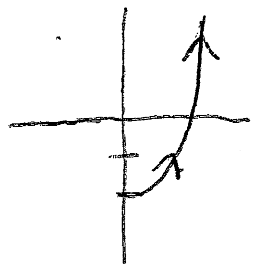
Sketch curve represented by the parametric equation

$x = \sqrt{t} \quad y = t - 2$

$t = x^2$

$\boxed{y = x^2 - 2}$

$x = \sqrt{t} \quad t \geq 0$



t	x	y
0	0	-2
1	1	-1
2	$\sqrt{2}$	0
3	$\sqrt{3}$	1
4	2	2

(4)

(Calculator) An object moving along a curve in the xy-plane has position $(x(t), y(t))$ with $\frac{dx}{dt} = \cos(t^2)$ and $\frac{dy}{dt} = \sin(t^3)$. At time $t = 0$, the object is at position $(4, 7)$. Where is the particle when $t = 2$?

(A) $\langle -0.564, 0.989 \rangle$ (B) $\langle 0.461, 0.452 \rangle$ (C) $\langle 3.346, 7.989 \rangle$

$\boxed{\text{(D) } \langle 4.461, 7.452 \rangle}$ (E) $\langle 5.962, 8.962 \rangle$

final position = initial position + displacement

$$x(2) = x(0) + \int_0^2 \cos(t^2)\,dt$$

$$x(2) = 4 + 0.461 = 4.461$$

$$y(2) = y(0) + \int_0^2 \sin(t^3)\,dt$$

$$= 7 + 0.452 = 7.452$$

$$(x(2), y(2)) = \langle 4.461, 7.452 \rangle$$

(5)

Find the arc length of the curve on the given interval

$x = \sqrt{t} \quad y = 3t - 1 \qquad 0 \leq t \leq 1$

$$\text{Length} = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\,dt$$

$x'(t) = \frac{1}{2} t^{-1/2} \qquad y'(t) = 3$

$$L = \int_0^1 \sqrt{\left(\frac{1}{2\sqrt{t}}\right)^2 + (3)^2}\,dt = \int_0^1 \sqrt{\frac{1}{4t} + 9}\,dt \approx \boxed{3.249}$$

(6)

Find the surface area generated by revolving the curve about the given axis:

$x = 2t,\ y = 4 - 2t$ x-axis $0 \le t \le 2$

$$S_x = 2\pi \int_a^b y \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

$x'(t) = 2$

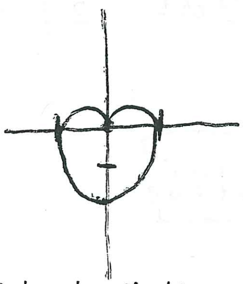
$y'(t) = -2$

$$S_x = 2\pi \int_0^2 (4-2t)\sqrt{2^2 + (-2)^2}\, dt$$

$$= 2\pi \sqrt{8} \int_0^2 4 - 2t\, dt = \sqrt{8}\left[4t - t^2\right]_0^2$$

$$\sqrt{8}(8-4) - (0) = \boxed{4\sqrt{8}} \cdot 2\pi$$

$$\boxed{8\pi\sqrt{8}}$$



(7)

nd points of horizontal and vertical tangency for $r = 1 - \sin\theta$

Horizontal tangent : set $\frac{dy}{d\theta} = 0$ vertical tangent: set $\frac{dx}{d\theta} = 0$

$y = r\sin\theta$

$y = (1 - \sin\theta)\sin\theta$

$y = \sin\theta - \sin^2\theta$

$\frac{dy}{d\theta} = \cos\theta - 2\sin\theta\cos\theta$

$0 = \cos\theta(1 - 2\sin\theta)$

$\cos\theta = 0$ | $1 - 2\sin\theta = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ | $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = r\cos\theta$

$x = (1 - \sin\theta)\cos\theta$

$x = \cos\theta - \sin\theta\cos\theta$

$\frac{dx}{d\theta} = -\sin\theta - [\cos\theta\cos\theta + \sin\theta(-\sin\theta)]$

$0 = -\sin\theta - \cos^2\theta + \sin^2\theta$

$0 = -\sin\theta - (1 - \sin^2\theta) + \sin^2\theta$

$= -\sin\theta - 1 + \sin^2\theta + \sin^2\theta$

$0 = 2\sin^2\theta - \sin\theta - 1$

$0 = (2\sin\theta + 1)(\sin\theta - 1)$

$\sin\theta = -\frac{1}{2}$ | $\sin\theta = 1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ | $\theta = \frac{\pi}{2}$

Horizontal tangents:

$(2, \frac{3\pi}{2})$

$(\frac{1}{2}, \frac{\pi}{6})$

$(\frac{1}{2}, \frac{5\pi}{6})$

Vertical tangents

$(\frac{3}{2}, \frac{7\pi}{6})$

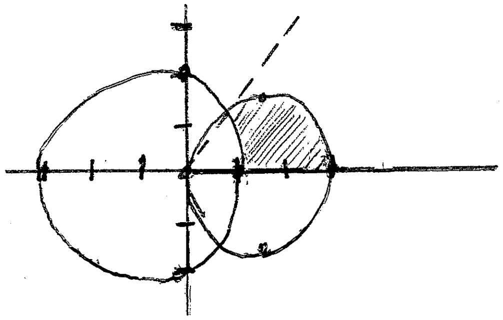
$(\frac{3}{2}, \frac{11\pi}{6})$

(8)

Find area of region

circle — dimpled limaçon

inside $r = 3\cos\theta$ and outside $r = 2 - \cos\theta$



Intersection:

$3\cos\theta = 2 - \cos\theta$

$4\cos\theta = 2$

$\cos\theta = \frac{1}{2}$

$\theta = \pi/3$

$$A = 2\left[\frac{1}{2}\int_0^{\pi/3}(3\cos\theta)^2\,d\theta - \frac{1}{2}\int_0^{\pi/3}(2-\cos\theta)^2\,d\theta\right]$$

$$A = 3\sqrt{3} \approx \boxed{5.196}$$