10.2-10.5 Review WS #2

(1)

(Calculator) A particle moves in the xy −plane so that its position at any time t is given by $x=\cos(5t)$ and $y=t^3$. What is the speed of the particle when $t=2$?

(2)

(Calculator) The position of a particle at time $t\geq 0$ is given by the parametric equations $x(t)=\frac{(t-2)^3}{3}+4$ and $y(t)=t^2-4t+4$.

(a) Find the magnitude of the velocity vector at $t=1$.

(b) Find the total distance traveled by the particle from $t=0$ to $t=1$.

(c) When is the particle at rest? What is its position at that time?

(3)

For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only (B) 1 only (C) 0 and 2/3 only (D) 0, 2/3, and 1 (E) No value

(4)

(Calculator) The path of a particle moving in the plane is defined parametrically as a function of time t by $x = \sin 2t$ and $y = \cos 5t$. What is the speed of the particle at $t = 2$?

(A) 1.130 (B) 3.018 (C) $\langle -1.307, 2.720 \rangle$ (D) $\langle 0.757, 0.839 \rangle$ (E) $\langle 1.307, 2.720 \rangle$

(5)

The distance traveled by a particle from $t = 0$ to $t = 4$ whose position is given by the vector $\vec{s}(t) = \langle t^2, t \rangle$ is given by

(A) $\int_0^4 \sqrt{4t+1}\,dt$ (B) $2\int_0^4 \sqrt{t^2+1}\,dt$ (C) $\int_0^4 \sqrt{2t^2+1}\,dt$ (D) $\int_0^4 \sqrt{4t^2+1}\,dt$ (E) $2\pi\int_0^4 \sqrt{4t^2+1}\,dt$

(6)

An object moving along a curve in the *xy*-plane has position $(x(t),\ y(t))$ at time t with the velocity vector

$v(t) = \left(\frac{1}{t+1},\ 2t \right)$. At time $t = 1$, the object is at (ln 2, 4).

(a) Find the position vector.

(b) Write an equation for the line tangent to the curve when $t = 1$.

(c) Find the magnitude of the velocity vector when $t = 1$.

(d) At what time $t > 0$ does the line tangent to the particle at $(x(t),\ y(t))$ have a slope of 12?

(7)

An object moving along a curve in the *xy*-plane has position $(x(t), y(t))$ at time t with

$\frac{dx}{dt} = \cos(e^t)$ and $\frac{dy}{dt} = \sin(e^t)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point (3, 2).

(a) Find the equation of the tangent line to the curve at the point where $t = 1$.

(b) Find the speed of the object at $t = 1$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.

(d) Find the position of the object at time $t = 2$.

10.2-10.5 Review WS #2

Key

(1)

(Calculator) A particle moves in the xy – plane so that its position at any time t is given by $x = \cos(5t)$ and $y = t^3$. What is the speed of the particle when $t = 2$?

$x'(t) = -5\sin(5t)$ $y'(t) = 3t^2$

$\vec{v}(t) = \langle -5\sin(5t), 3t^2 \rangle$

Speed $= \|\vec{v}(t)\| = \sqrt{x'(t)^2 + y'(t)^2}$

$\|\vec{v}(2)\| = \sqrt{25\sin^2(10) + 12^2} \approx \boxed{12.304}$

(2)

(Calculator) The position of a particle at time $t \geq 0$ is given by the parametric equations $x(t) = \frac{(t-2)^3}{3} + 4$ and $y(t) = t^2 - 4t + 4$.

(a) Find the magnitude of the velocity vector at $t = 1$. (speed)

$x'(t) = \frac{3(t-2)^2}{3}$ $y'(t) = 2t - 4$

$\|\vec{v}(1)\| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 1$.

Distance $= \int_0^1 \sqrt{(t-2)^4 + (2t-4)^2}\, dt \approx \boxed{3.815}$

(c) When is the particle at rest? What is its position at that time? (find when $x'(t) = y'(t) = 0$)

$x'(t) = (t-2)^2$ $y'(t) = 2t - 4$

$0 = (t-2)^2$ $0 = 2t - 4$

$t = 2$ $t = 2$ ✓

particle at rest when $t = 2$.

$\vec{p}(t) = (4, 0)$

(3)

For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only (B) 1 only (C) 0 and 2/3 only (D) 0, 2/3, and 1 (E) No value

*vertical tangent when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

$\frac{dx}{dt} = 3t^2 - 2t$

$0 = t(3t-2)$

$t = 0, 2/3$

$y'(0) \neq 0$, $y'(2/3) \neq 0$ } to ensure we don't have $\frac{0}{0}$

(4)

(Calculator) The path of a particle moving in the plane is defined parametrically as a function of time t by $x = \sin 2t$ and $y = \cos 5t$. What is the speed of the particle at $t = 2$?

(A) 1.130 (B) 3.018 (C) $\langle -1.307, 2.720 \rangle$ (D) $\langle 0.757, 0.839 \rangle$ (E) $\langle 1.307, 2.720 \rangle$

speed $= \sqrt{x'(t)^2 + y'(t)^2}$

$= \sqrt{(2\cos(2t))^2 + (5\sin(5t))^2}$

$= \sqrt{4\cos^2(4) + 25\sin^2(10)} \approx 3.017$

(5)

The distance traveled by a particle from $t = 0$ to $t = 4$ whose position is given by the vector $\vec{s}(t) = \langle t^2, t \rangle$ is given by

(A) $\int_0^4 \sqrt{4t+1}\,dt$ (B) $2\int_0^4 \sqrt{t^2+1}\,dt$ (C) $\int_0^4 \sqrt{2t^2+1}\,dt$ (D) $\int_0^4 \sqrt{4t^2+1}\,dt$ (E) $2\pi\int_0^4 \sqrt{4t^2+1}\,dt$

$\vec{v}(t) = \langle 2t, 1 \rangle$

Distance $= \int_0^4 \sqrt{(2t)^2 + (1)^2}\,dt$ D

(6)

An object moving along a curve in the *xy*-plane has position $(x(t),\, y(t))$ at time t with the velocity vector

$v(t)=\left(\frac{1}{t+1},\, 2t\right)$. At time $t = 1$, the object is at (ln 2, 4).

(a) Find the position vector. $x(t)=\int \frac{1}{1+t}\,dt = \ln|1+t| + c$

$x(1)=\ln|2|+c$

$\ln 2 = \ln 2 + c$

$0 = c$

$x(t)=\ln|1+t|+0$

$y(t)=\int 2t\,dt = \frac{2t^2}{2}+c$

$y(t)=t^2+c$

$y(1)=1+c \qquad 4=1+c \qquad c=3 \qquad y(t)=t^2+3$

$\vec{P}(t)=\langle \ln|1+t|,\, t^2+3 \rangle$

(b) Write an equation for the line tangent to the curve when $t = 1$.

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{2t}{\frac{1}{t+1}} = \frac{2}{\frac{1}{2}} = 4 \qquad$ point: $(\ln 2, 4)$

$y-y_1=m(x-x_1)$

$y-4=4(x-\ln 2)$

(c) Find the magnitude of the velocity vector when $t = 1$.

$\frac{1}{4}+4$

$\|\vec{v}(1)\| = \sqrt{\left(\frac{1}{t+1}\right)^2+(2t)^2} = \sqrt{\left(\frac{1}{2}\right)^2+2^2} = \sqrt{\frac{1}{4}+4} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$

(d) At what time $t > 0$ does the line tangent to the particle at $(x(t),\, y(t))$ have a slope of 12?

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 12$

$\frac{2t}{\frac{1}{t+1}} = 12$

$2t = \frac{12}{t+1}$

$2t(t+1)=12$

$2t^2+2t-12=0$

$2(t^2+t-6)=0$

$2(t+3)(t-2)=0$

$t=-3, 2$

$t=2$

(7)

An object moving along a curve in the *xy*-plane has position $(x(t), y(t))$ at time t with

$\frac{dx}{dt} = \cos(e^t)$ and $\frac{dy}{dt} = \sin(e^t)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point (3, 2).

(a) Find the equation of the tangent line to the curve at the point where $t = 1$.

$$m = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \left.\frac{\sin(e^t)}{\cos(e^t)}\right|_{t=1} = -0.451$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -0.451(x - 3)$$

(b) Find the speed of the object at $t = 1$.

$$\text{speed} = \sqrt{\sin^2(e^t) + \cos^2(e^t)} = \sqrt{\sin^2 e + \cos^2 e} = \sqrt{1} = 1$$

↳ pythagorean Identity

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.

$$\text{Distance} = \int_0^2 \sqrt{\sin^2(e^t) + \cos^2(e^t)}\,dt =$$

$$\int_0^2 1\,dt = t\Big]_0^2 = 2 - 0 = \boxed{2}$$

(d) Find the position of the object at time $t = 2$.

$$x(2) = x(1) + \int_1^2 \cos(e^t)\,dt = 3 - 0.104 = 2.896$$

$$y(2) = y(1) + \int_1^2 \sin(e^t)\,dt = 2 - 0.324 = 1.676$$

$$\boxed{\text{position} = (2.896, 1.676)}$$