BC Calculus Ch. 10.2 Notes Graphing Parametric Equations and Eliminating the Parameter

In Algebra, equations are graphed in two variables, x and y. Now we will graph equations with x, y, and t, or with x, y, and θ, where x and y are expressed independently in terms of t or θ. The third variable, t or θ is called the parameter, and the separate equation are called parametric equations.

Example 1:
Without a calculator, make a table, and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Verify on your calculator.

$$x = t^2 - 4 \text{ and } y = \frac{t}{2},\ -2 \le t \le 3$$

What do you notice about the graphs of $x = 4t^2 - 4$ and $y = t$, $-1 \le t \le 1.5$

What do you notice about the graphs of $x = 4(2\sin t + 1)^2 - 4$ and $y = 2\sin t + 1$, $-1.571 \le t \le 0.253$

While rectangular equations on restricted intervals show the ______________, parametric equations show

the __________________________, ________________________, and ___________________________.

Example 2:

Without a calculator, make a table, and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Verify on your calculator.

$$x=\frac{1}{\sqrt{t+1}},\ y=\frac{t}{t+1}$$

Example 3:

Without a calculator, make a table, and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Verify on your calculator.

$$x=2+3\cos t,\ y=-1+2\sin t$$

Key

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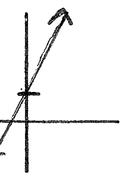
In Algebra, equations are graphed in two variables, x and y. Now we will graph equations with x, y, and t, or with x, y, and θ, where x and y are expressed independently in terms of t or θ. The third variable, t or θ is called the parameter, and the separate equation are called parametric equations.

• $y = 2x + 1$
(info on path of graph)



* Parametric equations provides more information about the graph: 1) speed at which graph is traced out
2) direction at which path is traced out

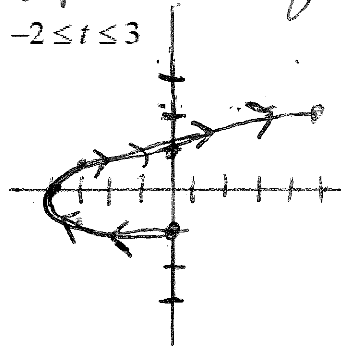
Example 1:
Without a calculator, make a table, and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Verify on your calculator. set of parametric equations

$$x = t^2 - 4 \text{ and } y = \frac{t}{2},\ -2 \le t \le 3$$

1 sec later

t	x	y
-2	0	-1
-1	-3	$-1/2$
0	-4	0
1	-3	$1/2$
2	0	1
3	5	$5/2$

moved left 3 units (faster)
up ½ unit



$y = t/2 \quad t = 2y$

$x = t^2 - 4 \rightarrow x = (2y)^2 - 4$

$x = 4y^2 - 4$

$y \in [-1, 1.5]$

Eliminate the parameter
(system of equation)
Solve for t.

same direction
same path, greater speed
b/c less time to cover the same path

What do you notice about the graphs of $x = 4t^2 - 4$ and $y = t$, $\boxed{-1 \le t \le 1.5}$

What do you notice about the graphs of $x = 4(2\sin t + 1)^2 - 4$ and $y = 2\sin t + 1$, $-1.571 \le t \le 0.253$

While rectangular equations on restricted intervals show the path, parametric equations show

the path, speed, and direction.

Example 2:

Without a calculator, make a table, and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Verify on your calculator.

*Look for shared (mutual) domain.

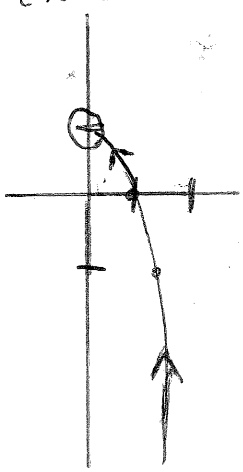
$$x = \frac{1}{\sqrt{t+1}}, \quad y = \frac{t}{t+1}$$

$t > -1$ $\quad$ $t \neq -1$

Domain: $t > -1$

t	x	y
$-1/2$	$\sqrt{2}$	-1
0	1	0
3	$1/2$	$3/4$
8	$1/3$	$8/9$
15	$1/4$	$15/16$

$\lim_{t \to \infty} \frac{t}{t+1} = 1$



$y = \frac{t}{t+1}$

$(t+1)y = t$

$ty + t = -y$

$t(y-1) = -y$

$t = \frac{-y}{y-1}$

$t = \frac{y}{1-y}$

$x = \frac{1}{\sqrt{\frac{y}{1-y} + 1}}$

$x^2 = \frac{1}{\frac{y}{1-y} + 1} \cdot \frac{(1-y)}{(1-y)}$

$x^2 = \frac{1-y}{y+1-y} = \frac{1-y}{1}$

$x^2 = 1 - y$

$\boxed{y = 1 - x^2}$

Restrict Domain

$x = \frac{1}{\sqrt{t+1}}$ $\quad t > -1$, so $\boxed{x > 0}$

Example 3:

Without a calculator, make a table, and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Verify on your calculator.

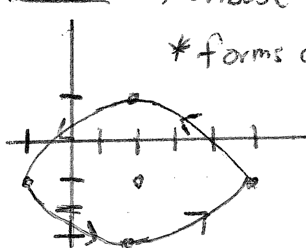
Domain: $\mathbb{R}$

$$x = 2 + 3\cos t, \quad y = -1 + 2\sin t$$

*choose unit circle t-values

*forms closed loop

t	x	y
0	5	-1
$\pi/6$	$2 + \frac{3\sqrt{3}}{2}$	0
$\pi/2$	2	1
π	-1	-1
$3\pi/2$	2	-3
2π	5	-1



center: $(2, -1)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-2)^2}{3^2} + \frac{(y+1)^2}{2^2} = 1$$

* Pythagorean Identity

$\cos^2 t + \sin^2 t = 1$

$\cos t = \frac{x-2}{3}$ $\quad$ $\sin t = \frac{y+1}{2}$

$$\left(\frac{x-2}{3}\right)^2 + \left(\frac{y+1}{2}\right)^2 = 1$$

$$\boxed{\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1}$$

rectangular equation of the path.