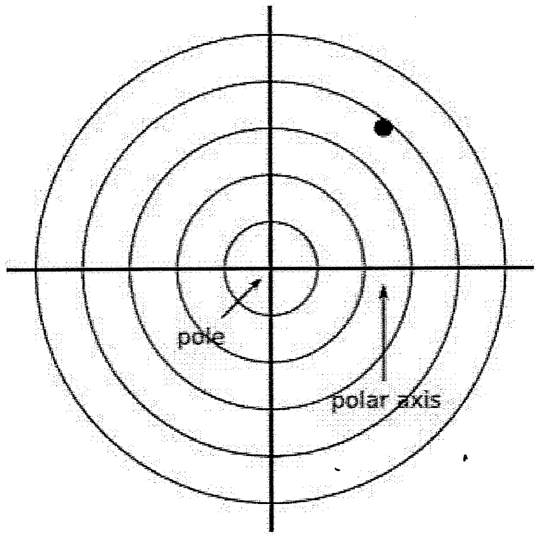
BC Calculus Ch. 10.4a Notes Graphing Polar Equations

A rectangular coordinate system is only one way to navigate through a Euclidean plane. Such coordinates, (x, y), known as **rectangular coordinates**, are useful for expressing functions of y in terms of x. Curves that are not functions are often more easily expressing in an alternative coordinate system called **polar coordinates.**



In a polar coordinate system, we still have the traditional x- and y- axes. The intersection of these axes, the old origin, is called the **pole**. Similar to navigating on the Unit Circle, we can now get to any point in 2-D space by specifying an independent choice of an angle, θ, from the initial ray, **polar axis**, then walking out along that terminal ray a specified amount, r, in either direction.

Although the angle is the independent variable, we express the point in the polar plane as (r, θ). The point to the right would have coordinates of $\left(4, \frac{\pi}{4}\right)$.

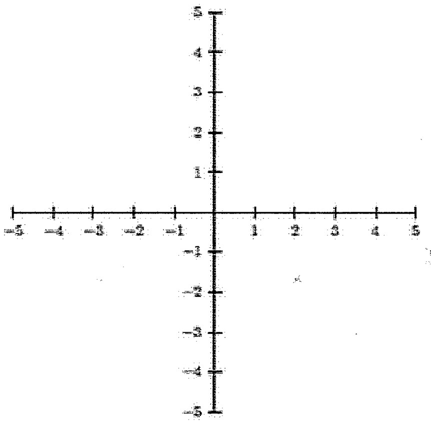
Example 1:
Find several other equivalent polar coordinates for the point shown above, then find the equivalent rectangular coordinate.

Why use polar coordinates? Graphs that aren't functions in rectangular form $f(x)$ can still be functions in polar form $r(\theta)$. Some of these curves can be quite elaborate and are more easily expressed as polar, rather than rectangular equations, as the following calculator exploration will demonstrate.

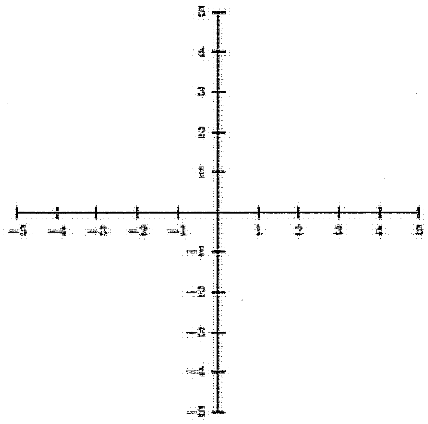
Example 2:

Put your graphing calculator in POLAR mode and RADIAN mode. Graph the following equations on your calculator, sketch the graphs on this sheet, and answer the questions.

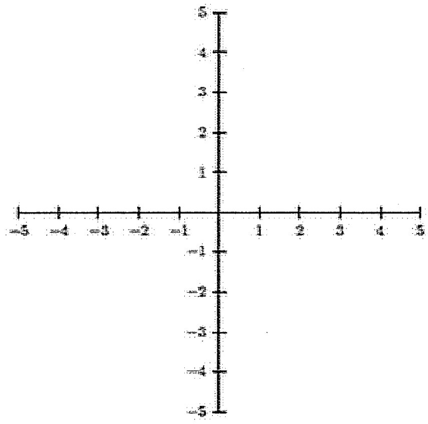
$r = cos\theta$



$r = 3cos\theta$



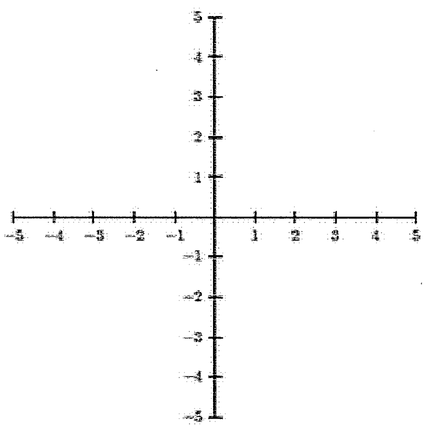
$r = 4sin\theta$



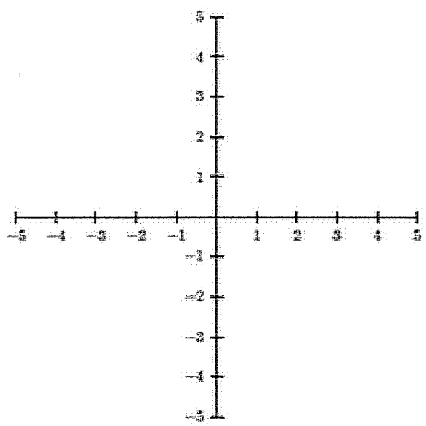
What do you notice about the above graphs?

Example 3a:

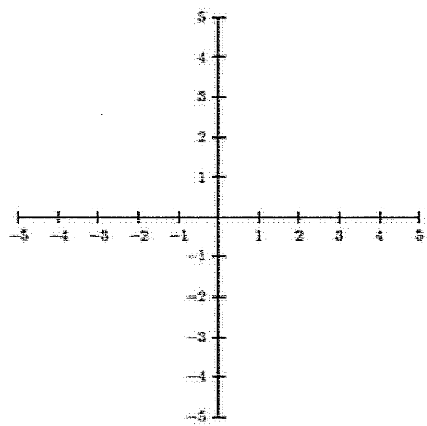
$r = 2 + 2cos\theta$



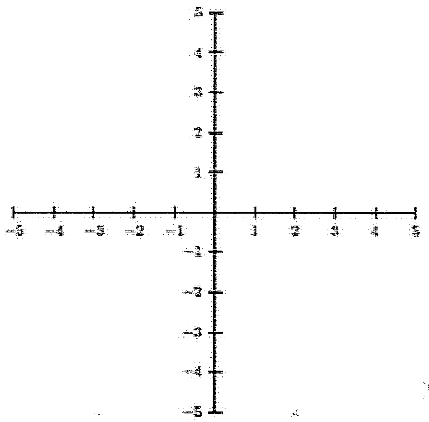
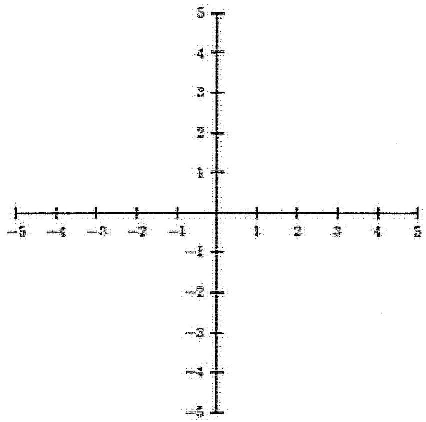
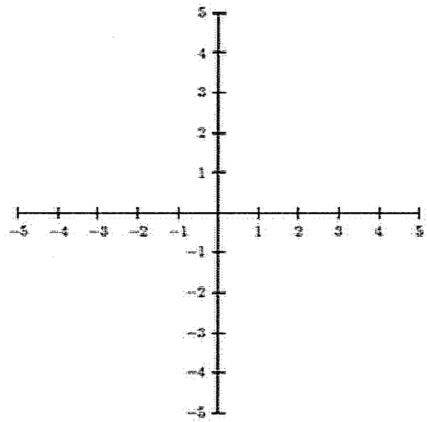
$r = 1 + 2cos\theta$



$r = 2 + cos\theta$

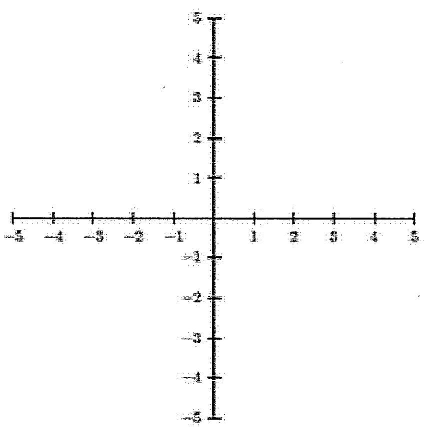
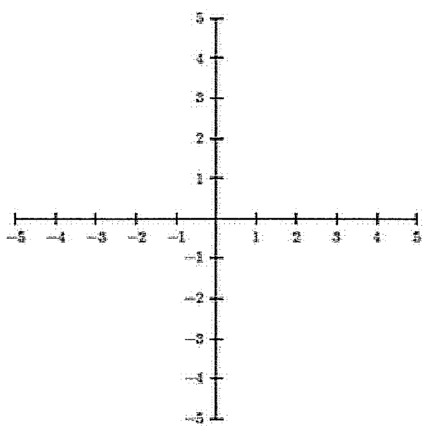
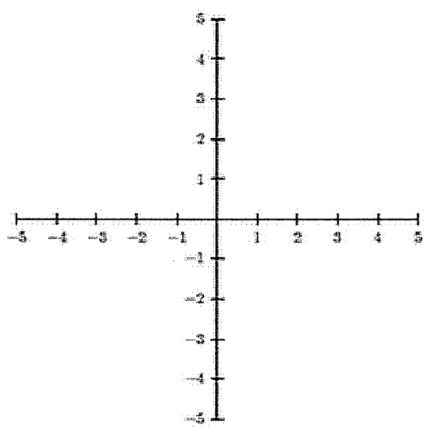


Example 3b:

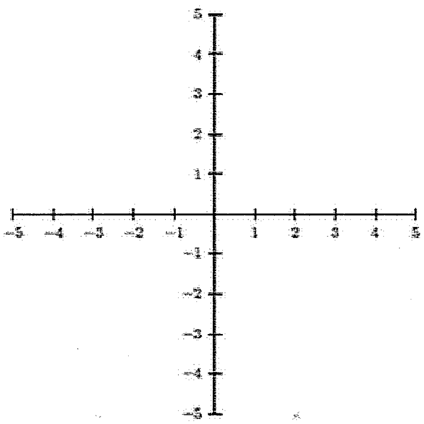
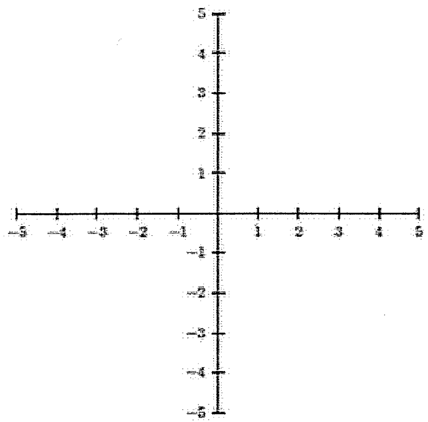
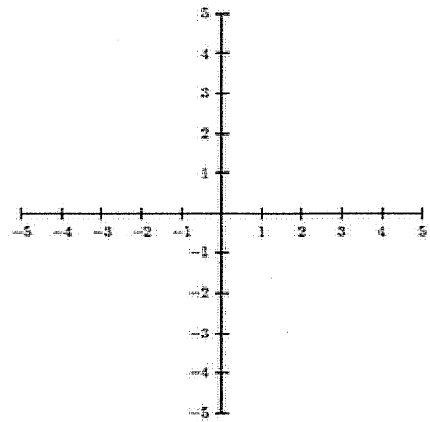
$r = 2 + 2sin\theta$	$r = 1 + 2sin\theta$	$r = 2 + sin\theta$
		

Which graphs go through the pole?
Which ones do not go through the pole?
Which ones have an inner loop?
What causes these things to happen? (Hint: Go to FORMAT and set your calculator to see the Polar Graphing Coordinates when you trace.)

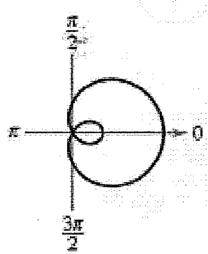
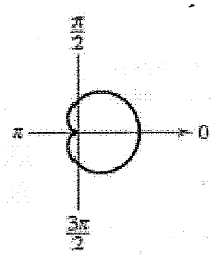
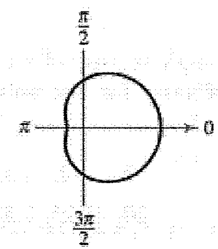
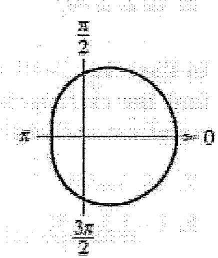
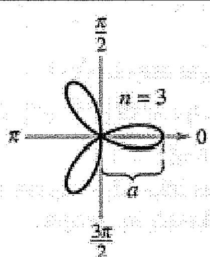
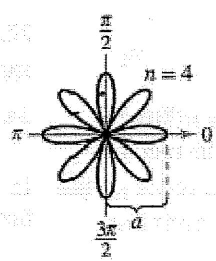
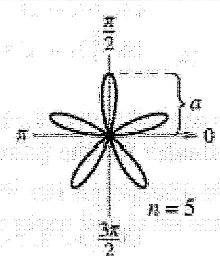
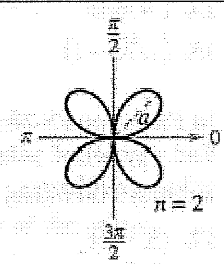
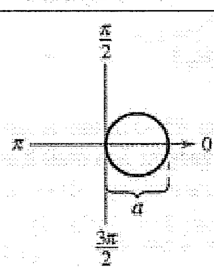
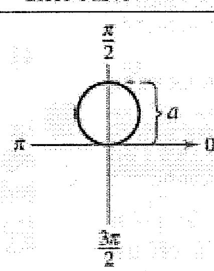
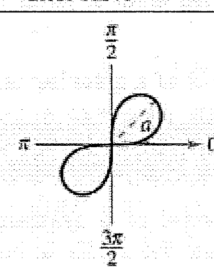
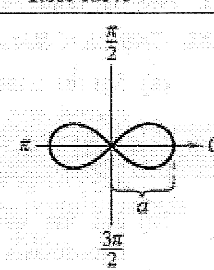
Example 4:

$r = 3cos3\theta$	$r = 2sin5\theta$	$r = 4cos7\theta$
		

Example 5:

$r = 3cos2\theta$	$r = 2sin4\theta$	$r = 4cos6\theta$
		

What do you notice about the above graphs?

<u>Limacons</u> r = a ± b cos θ r = a ± b sin θ (a > 0, b > 0)	 $\frac{a}{b} < 1$ Limaçon with inner loop	 $\frac{a}{b} = 1$ Cardioid (heart-shaped)	 $1 < \frac{a}{b} < 2$ Dimpled limaçon	 $\frac{a}{b} \geq 2$ Convex limaçon
<u>Rose Curves</u> n petals if n is odd 2n petals if n is even	 $r = a\cos n\theta$ Rose curve	 $r = a\cos n\theta$ Rose curve	 $r = a\sin n\theta$ Rose curve	 $r = a\sin n\theta$ Rose curve
<u>Circles</u> and **<u>Lemniscates</u>**	 $r = a\cos\theta$ Circle	 $r = a\sin\theta$ Circle	 $r^2 = a^2\sin 2\theta$ Lemniscate	 $r^2 = a^2\cos 2\theta$ Lemniscate

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Key

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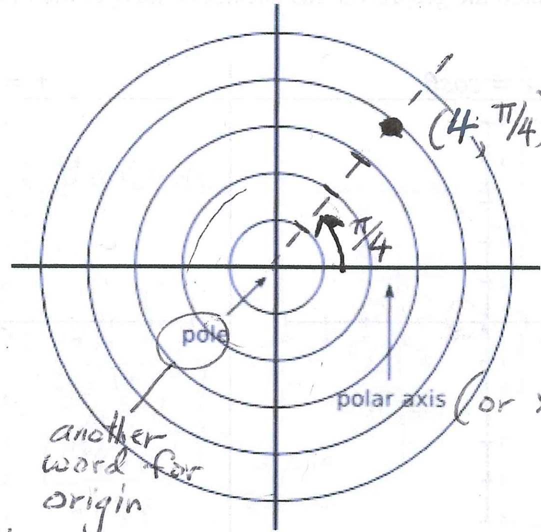
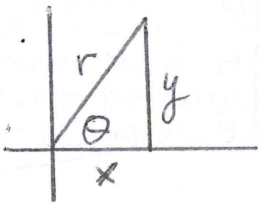


In a polar coordinate system, we still have the traditional x- and y- axes. The intersection of these axes, the old origin, is called the **pole**. Similar to navigating on the Unit Circle, we can now get to any point in 2-D space by specifying an independent choice of an angle, θ, from the initial ray, **polar axis**, then walking out along that terminal ray a specified amount, r, in either direction.

Although the angle is the independent variable, we express the point in the polar plane as (r, θ) (independent variable). The point to the right would have coordinates of $\left(4, \frac{\pi}{4}\right)$.

↳ 4 units out along the angle π/4 from the polar axis.



$\sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$

$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

Example 1: (within 1 rotation from polar axis)

Find several other equivalent polar coordinates for the point shown above, then find the equivalent rectangular coordinate.

Equivalent polar coordinates to $(4, \pi/4)$ $(r, \theta) =$

1) $(-4, 5\pi/4)$
2) $(4, -7\pi/4)$
3) $\left(-4, -\frac{3\pi}{4}\right)$

equivalent rectangular coordinate

$x = r\cos\theta = 4\cos\pi/4 = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$

$y = r\sin\theta = 4\sin\pi/4 = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$

$(x, y) = (2\sqrt{2}, 2\sqrt{2})$

Why use polar coordinates? Graphs that aren't functions in rectangular form $f(x)$ can still be functions in polar form $r(\theta)$. Some of these curves can be quite elaborate and are more easily expressed as polar, rather than rectangular equations, as the following calculator exploration will demonstrate.

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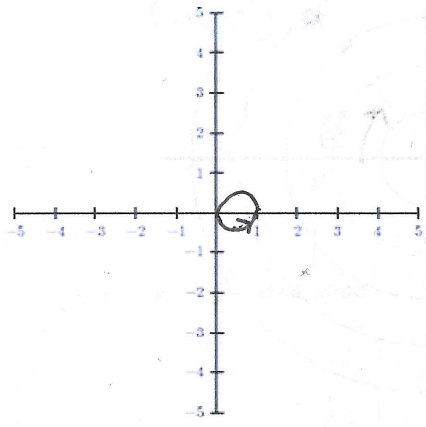
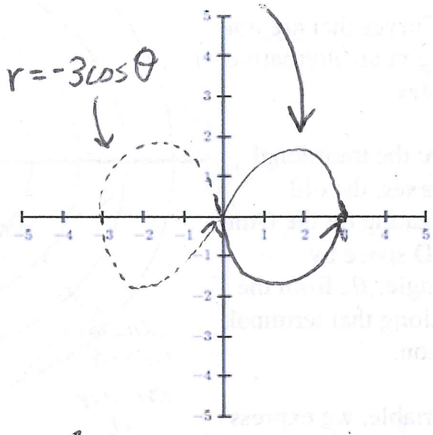


θ	r
0	1
π/2	0
π	-1

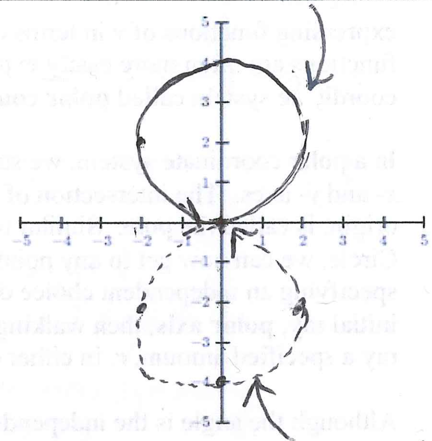
* Traces in CCW and 1 rotation in [0, π]

$r = 3\cos\theta$



θ	r
0	3
π/2	0
π	-3

$r = 4\sin\theta$



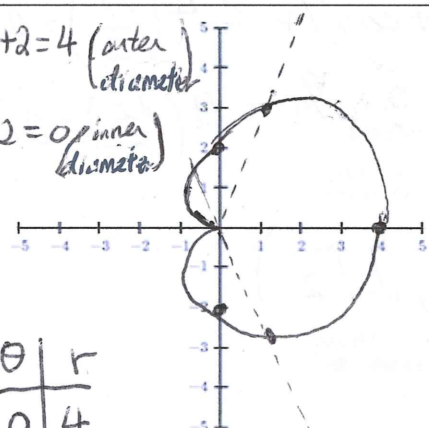
θ	r
0	0
π/2	4
π	0

What do you notice about the above graphs?

1) diameter of circle matches coefficient
2) traces out in CCW, 1 rotation [0, π]
3) cosine graph (symmetry about x-axis), sine graph (symmetry about y-axis)

Example 3a:

$r = 2 + 2\cos\theta$

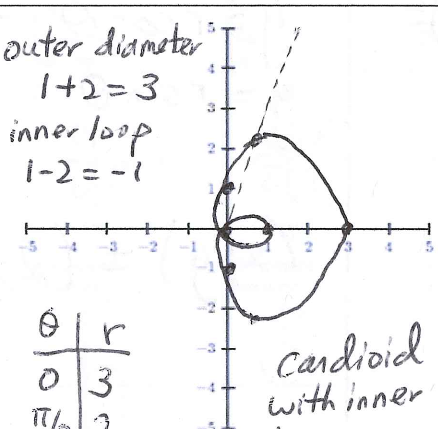


θ	r
0	4
π/3	3
π/2	2
2π/3	1
π	0

cardioid [0, 2π]

π | 0 → polar zero (origin) at θ = π

$r = 1 + 2\cos\theta$

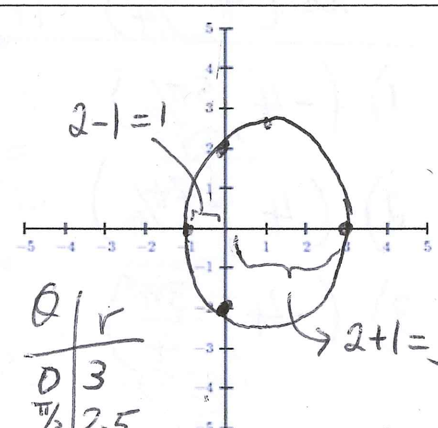


θ	r
0	3
π/3	2
π/2	1
2π/3	0
π	-1
4π/3	0

cardioid with inner loop (limaçon) [0, 2π]

→ 2 polar zeros at θ = 2π/3, 4π/3

$r = 2 + \cos\theta$



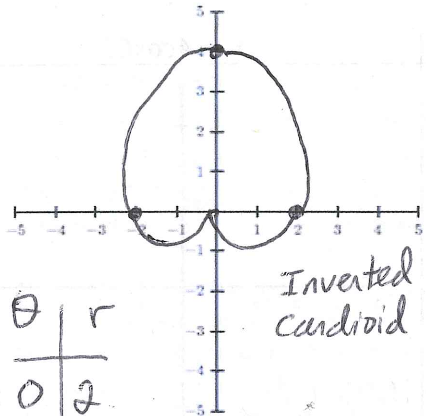
θ	r
0	3
π/3	2.5
π/2	2
2π/3	1.5
π	1

dimpled limaçon

* No polar zeros.

Example 3b:

$r = 2 + 2sin\theta$

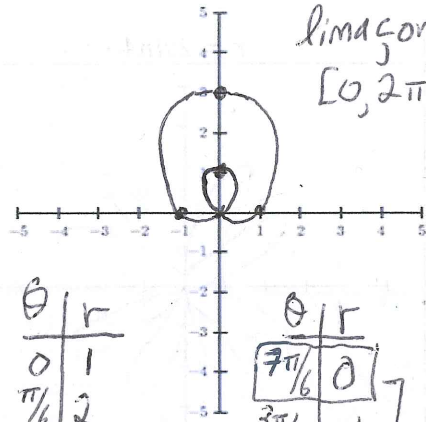


Inverted cardioid

θ	r
0	2
$\pi/2$	4
π	2
$3\pi/2$	0
2π	2

→ polar zero at $\theta = \frac{3\pi}{2}$

$r = 1 + 2sin\theta$



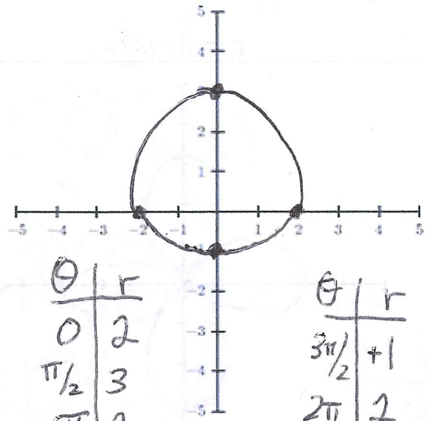
limaçon $[0, 2\pi]$

θ	r
0	1
$\pi/6$	2
$\pi/2$	3
π	1

θ	r
$7\pi/6$	0
$3\pi/2$	-1
$11\pi/6$	0
2π	1

2 polar zeros

$r = 2 + sin\theta$



θ	r
0	2
$\pi/2$	3
π	2

θ	r
$3\pi/2$	+1
2π	2

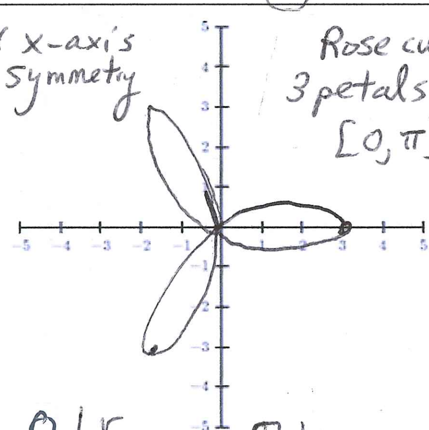
Which graphs go through the pole? cardioid (once), limaçon (twice), dimpled limaçon (none)
Which ones do not go through the pole? dimpled limaçon
Which ones have an inner loop? limaçon
What causes these things to happen? (Hint: Go to FORMAT and set your calculator to see the Polar Graphing Coordinates when you trace.)

Example 4:

petal length

odd # indicates the number of petals

$r = 3cos3\theta$



* x-axis symmetry

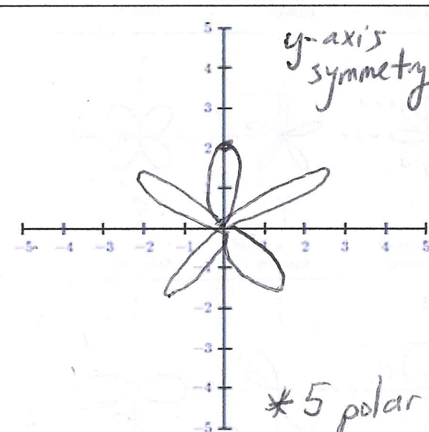
Rose curves
3 petals
$[0, \pi]$

θ	r
0	3
$\pi/6$	0
$\pi/3$	-3
$\pi/2$	0

θ	r
$2\pi/3$	3
$5\pi/6$	0
π	-3

3 polar zeros

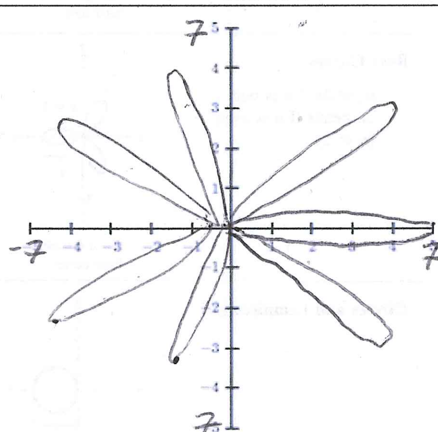
$r = 2sin5\theta$



y-axis symmetry

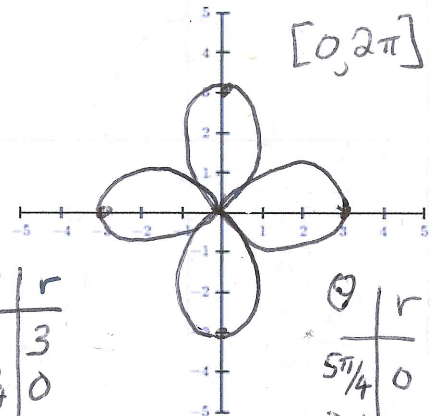
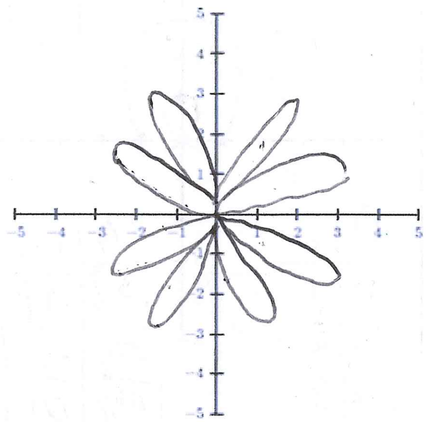
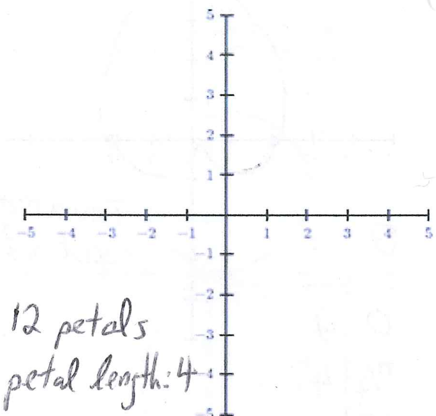
* 5 polar zeros

$r = 4cos7\theta$



* 7 polar zeros
* y-axis symmetry

Example 5: petal length | twice the number of petals (4)

$r = 3\cos 2\theta$	$r = 2\sin 4\theta$	$r = 4\cos 6\theta$
		

[0, 2π]

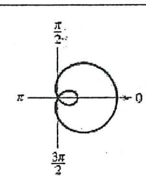
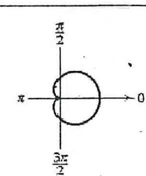
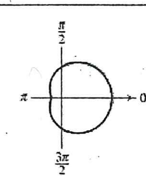
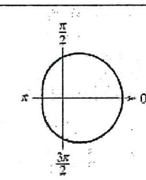
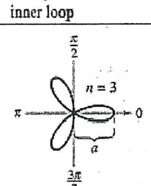
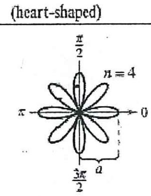
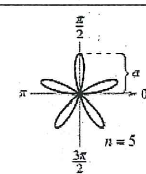
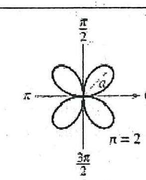
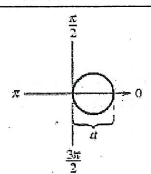
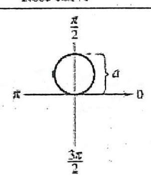
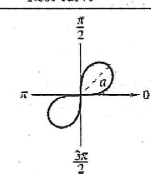
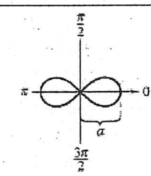
θ	r
0	3
π/4	0
π/2	-3
3π/4	0
π	3

θ	r
5π/4	0
3π/2	-3
7π/4	0
2π	3

12 petals
petal length: 4
x-axis symmetry
12 polar zeros
[0, 2π]

What do you notice about the above graphs?

n petals if n is odd [0, π]
2n petals if n is even [0, 2π]

Limacons r = a ± b cos θ r = a ± b sin θ (a > 0, b > 0)	 $\frac{a}{b} < 1$ Limaçon with inner loop	 $\frac{a}{b} = 1$ Cardioid (heart-shaped)	 $1 < \frac{a}{b} < 2$ Dimpled limaçon	 $\frac{a}{b} \geq 2$ Convex limaçon
Rose Curves n petals if n is odd 2n petals if n is even	 $r = a\cos n\theta$ Rose curve	 $r = a\cos n\theta$ Rose curve	 $r = a\sin n\theta$ Rose curve	 $r = a\sin n\theta$ Rose curve
Circles and **Lemniscates**	 $r = a\cos\theta$ Circle	 $r = a\sin\theta$ Circle	 $r^2 = a^2\sin 2\theta$ Lemniscate	 $r^2 = a^2\cos 2\theta$ Lemniscate