THEOREM 12.3 Position Function for a Projectile

Neglecting air resistance, the path of a projectile launched from an initial height h with initial speed v_0 and angle of elevation θ is described by the vector function

$$\mathbf{r}(t) = (v_0 \cos\theta)t\mathbf{i} + \left[h + (v_0 \sin\theta)t - \frac{1}{2}gt^2\right]\mathbf{j}$$

where g is the gravitational constant.

1. A baseball is hit from a height of 2.5 feet above the ground with an initial velocity of 140 ft/s and at angle of 22° above the horizontal.
   a. Determine whether the ball will clear a 10-foot high fence located 375 feet from home plate
   b. Determine the maximum height of the ball.
   c. Determine the distance (horizontal) from home plate when the ball hits the ground.

2.

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

BC Calculus **Ch. 10.4 – 12.3 Review WS #1**

Key

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1. A baseball is hit from a height of 2.5 feet above the ground with an initial velocity of 140 ft/s and at angle of 22° above the horizontal.
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a) $r(t) = 140(\cos 22)t\,i + (2.5 + (140\sin 22)t - 16t^2)\,j$

* Set $x(t) = 375$, $140(\cos 22)t = 375$ $\quad t \approx 2.889$ secs.

* Find $y(2.889) = 2.5 + 140\sin 22(2.889) - 16(2.889)^2 = \boxed{20.47 \text{ ft}}$ (clears 10 ft fence)

b) Max height when $y'(t) = 0$

$y'(t) = 140\sin 22 - 32t = 0 \quad t = 1.639$ seconds.

$\boxed{y(1.639) = 45.476 \text{ ft. (max height)}}$

c) * set $y(t) = 0 \quad t \approx 3.325$ seconds

Horizontal distance is $x(3.325) \approx \boxed{431.604 \text{ ft from home plate}}$

2.

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(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $\underline{2t + 1}$. Find the acceleration vector of the object at time $t = 4$.

$\frac{dy}{dx} = 2t + 1$

a) * final position = initial + displacement

$$x(4) = x(2) + \int_2^4 x'(t)\,dt = \boxed{7.132}$$

b) $$\left.\frac{dy}{dx}\right| = \left.\frac{dy/dt}{dx/dt}\right|_{t=2} = \frac{-7}{3+\cos(2^2)} = -2.983$$

slope: $m = -2.983$

point: $(1, 8)$

$$\boxed{y - 8 = -2.983(x-1)}$$

c) $$\text{Speed} = \sqrt{x'(t)^2 + y'(t)^2}$$

$$\|v(2)\| = \sqrt{x'(2)^2 + y'(2)^2} = \sqrt{(3+\cos 4)^2 + (7)^2} = \boxed{7.383}$$

d) $x''(4) = \underline{2.303}$ $\longrightarrow$ (math 8 nderiv $(3+\cos(x^2), x, 4)$)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$2t+1 = \frac{\frac{dy}{dt}}{3+\cos(t)^2}$$

$$\frac{dy}{dt} = (2t+1)[3+\cos(t^2)] \longrightarrow \text{nderiv}((2x+1)(3+\cos x^2), x, 4) = 24.814$$

so $y''(4) = \underline{24.814}$

The acceleration vector at $t=4$ is $\langle 2.303, 24.814 \rangle$

Question 3

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

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(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

(a) $x(4) = x(2) + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt$

$= 1 + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt = 7.132 \text{ or } 7.133$

$3 : \begin{cases} 1 : \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{cases}$

$\frac{dy}{dx} = 2t + 1$

(b) $\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=2} = \frac{-7}{3 + \cos 4} = -2.983$

$y - 8 = -2.983(x - 1)$

$2 : \begin{cases} 1 : \text{finds } \left.\frac{dy}{dx}\right|_{t=2} \\ 1 : \text{equation} \end{cases}$

$x''(4) =$ ______

$y''(4) =$

(c) The speed of the object at time $t = 2$ is $\sqrt{(x'(2))^2 + (y'(2))^2} = 7.382 \text{ or } 7.383$.

1 : answer

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

(d) $x''(4) = 2.303$

$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t + 1)\left(3 + \cos\left(t^2\right)\right)$

$y''(4) = 24.813 \text{ or } 24.814$

The acceleration vector at $t = 4$ is $\langle 2.303, 24.813 \rangle$ or $\langle 2.303, 24.814 \rangle$.

$3 : \begin{cases} 1 : x''(4) \\ 1 : \frac{dy}{dt} \\ 1 : \text{answer} \end{cases}$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$=$