BC Calculus Ch. 10.4-12.3 Review 2

1)

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dy}{dt} = 2 + \sin\left(e^t\right)$. The derivative of $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point $(4,5)$.

a) Find the y-coordinate of the position at time $t = 1$.

b) At time t = 3, the graph is tangent to line y = -1.8x + 12. Find the value of dx/dt when t = 3

c) Find the speed of the object at time t = 3

2)

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = \cos\left(e^t\right)$ and $\frac{dy}{dt} = \sin\left(e^t\right)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point (3, 2).

(a) Find the equation of the tangent line to the curve at the point where $t = 1$.
(b) Find the speed of the object at $t = 1$.
(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.
(d) Find the position of the object at time $t = 2$.

3.)

Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

4.) Find area of the inner loop created by r = 1+ $2cos\theta$

Key

BC Calculus Ch. 10.4-12.3 Review 2

An object moving along a curve in the xy-plane has position $\langle x(t), y(t)\rangle$ at time t with $\frac{dy}{dt} = 2 + \sin\left(e^t\right)$. The derivative of $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point $(4,5)$.

a) Find the y-coordinate of the position at time $t = 1$.

$\frac{dy}{dx} = -1.8$

b) At time t = 3, the graph is tangent to line y = -1.8x + 12. Find the value of dx/dt when t = 3

c) Find the speed of the object at time t = 3

a) * final position = initial + displacement : $y(1) = y(3) + \int_3^1 2 + \sin(e^t)\,dt$

$= 5 + -3.731 = \boxed{1.269}$

b) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2+\sin(e^t)}{dx/dt} = -1.8$, $\left.\frac{dx}{dt}\right|_{t=3} = \frac{2+\sin(e^t)}{-1.8} = \boxed{-1.636}$

c) $\text{Speed}\Big|_{t=3} = \sqrt{(-1.636)^2 + (2+\sin(e^3))^2} = \sqrt{(-1.636)^2 + (2.945)^2} = \sqrt{11.349} = \boxed{3.369}$

2.

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = \cos\left(e^t\right)$ and $\frac{dy}{dt} = \sin\left(e^t\right)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point (3, 2).

(a) Find the equation of the tangent line to the curve at the point where $t = 1$.
(b) Find the speed of the object at $t = 1$.
(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.
(d) Find the position of the object at time $t = 2$.

a) when $t = 1$, $\frac{dy}{dx} = \left.\frac{\sin(e^t)}{\cos(e^t)}\right|_{t=1} = -0.451$

slope: $m = -0.451$

point: $(3,2)$

$\boxed{y - 2 = -0.451(x-3)}$

b) $\text{Speed}\Big|_{t=1} = \sqrt{[\cos(e^t)]^2 + [\sin(e^t)]^2} = \boxed{1}$

c) $\text{Distance} = \int_0^2 \sqrt{(\cos e^t)^2 + (\sin e^t)^2}\,dt = \boxed{2}$

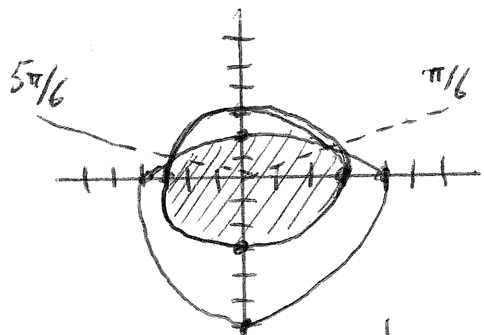
d) $x(2) = x(1) + \int_1^2 \cos(e^t)\,dt = 2.896$

$y(2) = y(1) + \int_1^2 \sin(e^t)\,dt = 1.676$

$\boxed{\text{position} = (2.896, 1.676)}$

3.

Let S be the shaded region that is inside the graph of $r = 3$ (circle) and also inside the graph of $r = 4 - 2\sin\theta$ (dimpled limaçon).
Find the area of S.



$3 = 4 - 2\sin\theta$

$2\sin\theta = 1$

$\sin\theta = 1/2$

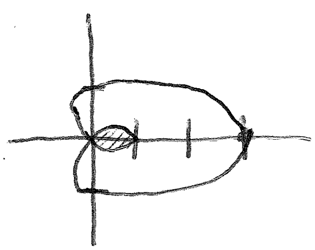
$\theta = \pi/6,\ 5\pi/6$

*use symmetry

$$A = 2\left[\underbrace{\frac{1}{2}\int_0^{\pi/6}(3)^2\,d\theta}_{\text{circle portion of arc}} + \underbrace{\frac{1}{2}\int_{\pi/6}^{\pi/2}(4-2\sin\theta)^2\,d\theta}_{\text{limacon portion of arc}}\right] + \underbrace{\frac{1}{2}\pi(3)^2}_{\text{semi circle (bottom half of shaded region)}}$$

$$4.7124 + 5.8592 + 14.137 = \boxed{24.708}$$

4. Find area of the inner loop created by r = 1+ $2cos\theta$



* Find polar zeros.

$1 + 2\cos\theta = 0 \qquad \theta = 2\pi/3,\ 4\pi/3$

$2\cos\theta = -1$

$\cos\theta = -1/2$

$$A = \frac{1}{2}\int_{2\pi/3}^{4\pi/3}\left[1+2\cos\theta\right]^2 d\theta = \pi - \frac{3\sqrt{3}}{2} \approx \boxed{0.544}$$