

A vector valued function or vector function is given by

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$$

OR

$$\mathbf{r}(t) = \langle f(t), g(t) \rangle$$

For a 2D curve or plane curve

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

OR

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$$

For a 3D curve or space curve

The curves are traced out by the terminal points of the vectors formed by the function values of  $f$ ,  $g$ , and  $h$

A **Vector Function** (can be expressed as vector form or component form)

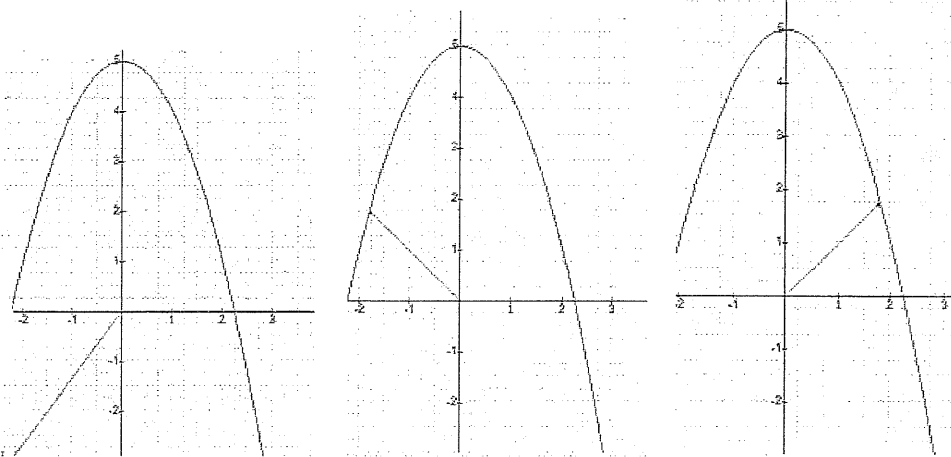
For a given value of  $t$ , a vector-valued function returns a vector

Graph in blue (plane curve)

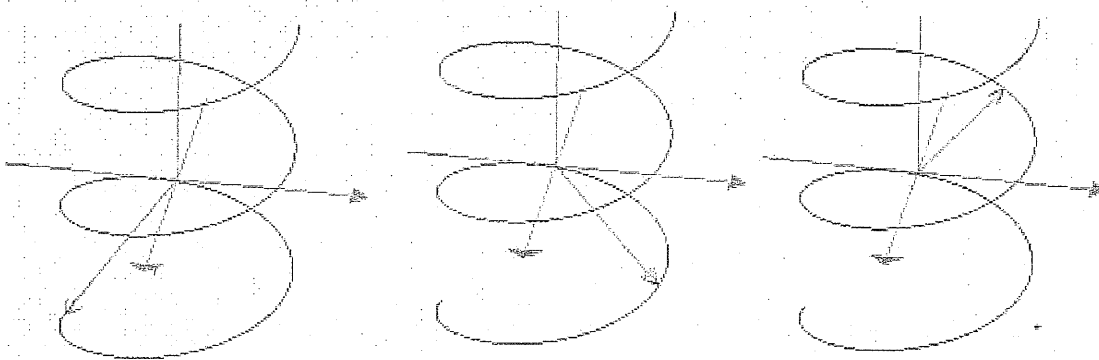
Vectors that trace out this curve from the vector value function (in red)

The direction in which the curve is traced is called the orientation (we can show direction by drawing arrows on the curve)

The terminal point from the vector traces out the plane curve. (starts from  $t = -3$  and watch as  $t$  increases)



Now for a space curve generated by a vector valued function (with  $t$  increasing from  $t = -8$  to  $t = 8$ )



When we evaluated the vector-valued function for different values of  $t$ , we were finding individual vectors that trace out this curve

As  $t$  increases, the point  $(x, y, z)$  spirals up the cylinder to produce a helix.

**Example 1:**  $r(t) = ti + (-t^2 + 5)j$

- a) Evaluate the vector value function for each value of t
- b) Write an equation for the function in rectangular form
- c) Graph the vector value function

a)  $r(0) =$

$r(2) =$

$r(-3) =$

**Example 2:**  $r(t) = 2\cos(t)i + 2\sin(t)j + tk$

Evaluate the vector value function for each value of t

Graph the vector value function

$r(0) =$

$r(\pi/4) =$

$r(\frac{2\pi}{3}) =$

The domain of a vector-valued function  $r$  is considered to be the intersection of the domains of the component functions  $f$ ,  $g$ , and  $h$ .

Key

A vector valued function or vector function is given by

$$r(t) = f(t)i + g(t)j$$

OR

$$r(t) = \langle f(t), g(t) \rangle$$

*← component form*

For a 2D curve or plane curve

$$r(t) = f(t)i + g(t)j + h(t)k$$

OR

$$r(t) = \langle f(t), g(t), h(t) \rangle$$

For a 3D curve or space curve

The curves are traced out by the terminal points of the vectors formed by the function values of f, g, and h

A **Vector Function** (can be expressed as vector form or component form)

For a given value of t, a vector-valued function returns a vector

Graph in blue (plane curve)

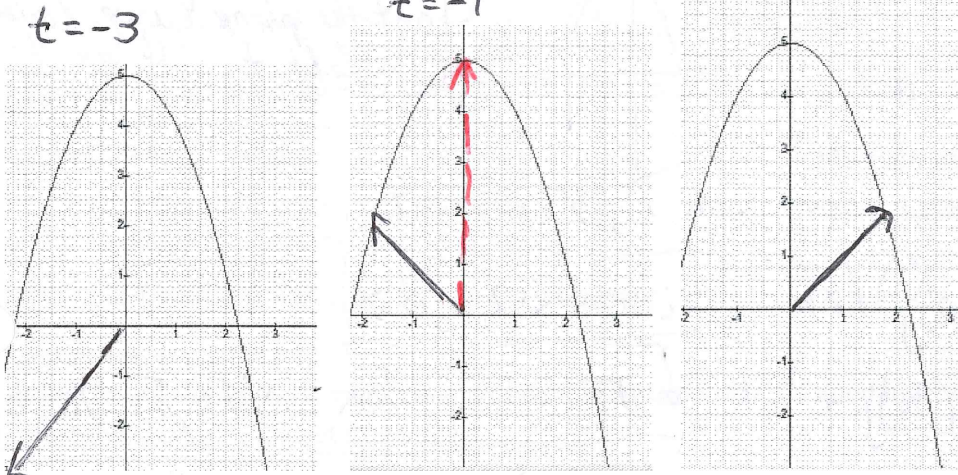
Vectors that trace out this curve from the vector value function (in red)

The direction in which the curve is traced is called the orientation (we can show direction by drawing arrows on the curve)

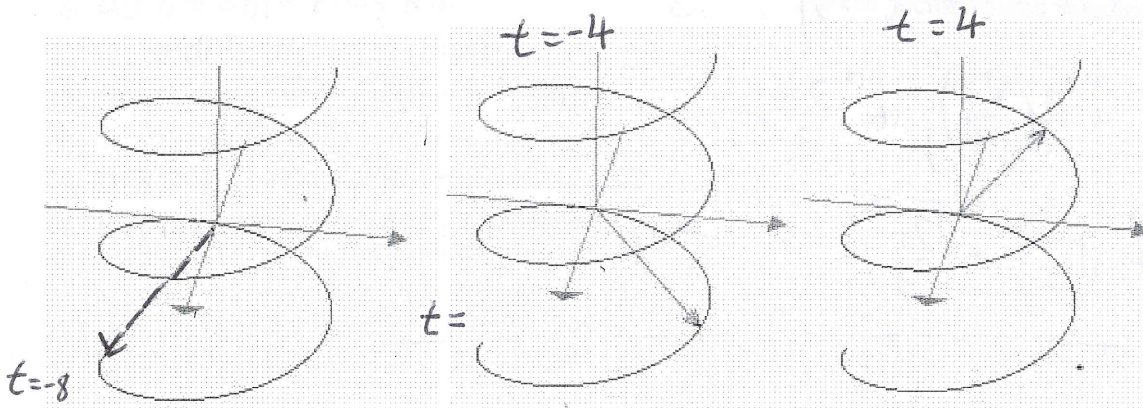
The terminal point from the vector traces out the plane curve. (starts from t = -3 and watch as t increases)

t = 2

*produced by vector valued function*



Now for a space curve generated by a vector valued function (with t increasing from t = -8 to t = 8)



When we evaluated the vector-valued function for different values of t, we were finding individual vectors that trace out this curve

As t increases, the point (x,y,z) spirals up the cylinder to produce a helix.

*\*curves are traced out in a specific direction called orientation.*

*show orientation by placing arrow on curves*

**Example 1:**  $r(t) = ti + (-t^2 + 5)j$

a) Evaluate the vector value function for each value of  $t$

b) Write an equation for the function in rectangular form

c) Graph the vector value function *\*replace  $t$  with zero*

$$a) r(0) = 0i + (-0^2 + 5)j = 5j = \langle 0, 5 \rangle$$

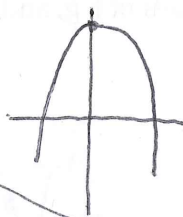
$$r(2) = 2i + (-(2)^2 + 5)j = 2i + j = \langle 2, 1 \rangle$$

$$r(-3) = -3i + (-(-3)^2 + 5)j = -3i - 4j = \langle -3, -4 \rangle$$

b)  $x = t$   
 $y = -t^2 + 5$

$$y = -x^2 + 5$$

c)



This is one vector generated by the vector-valued function where the terminal point of this vector will be a point on the plane curve

\*As  $t$  increases, the terminal point from vector (produced by vector-valued function) traces out the plane curve. (orientation is left to right as  $t$  increases)

**Example 2:**  $r(t) = 2\cos(t)i + 2\sin(t)j + tk$

Evaluate the vector value function for each value of  $t$

Graph the vector value function *\*space curve with  $x, y, z$  component*

$$r(0) = 2\cos(0)i + 2\sin(0)j + 0k = 2i = \langle 2, 0, 0 \rangle$$

$$r(\pi/4) = 2\cos(\pi/4)i + 2\sin(\pi/4)j + \pi/4k = \sqrt{2}i + \sqrt{2}j + \pi/4k = \langle \sqrt{2}, \sqrt{2}, \pi/4 \rangle$$

$$r(2\pi/3) = 2\cos(2\pi/3)i + 2\sin(2\pi/3)j + \frac{2\pi}{3}k$$

$$= 2(-1/2)i + 2(\frac{\sqrt{3}}{2})j + \frac{2\pi}{3}k$$

$$= -i + \sqrt{3}j + \frac{2\pi}{3}k = \langle -1, \sqrt{3}, \frac{2\pi}{3} \rangle$$

The domain of a vector-valued function  $r$  is considered to be the intersection of the domains of the component functions  $f$ ,  $g$ , and  $h$ .