BC Calculus   Ch. 8.7 Notes   L'Hopital's Rule

L'Hôpital's Rule (or Bernoulli's Rule)

If $\lim_{x\to a}\frac{f(x)}{g(x)}$ yields either of the indeterminate forms $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$.

The rule works great, but it only works with the two forms $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$. There are other indeterminate forms including 0^0, 1^∞, $\infty-\infty$, $0\cdot\infty$, and ∞^0. We can still use the rule, but we have to first convert them to $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$.

1.If Indeterminate form is 0^0, 1^∞, or ∞^0 → rewrite as equation and use Log Differentiation

2. If Indeterminate form is $\infty-\infty$ → find common denominator, which will get the expression into a single quotient, ready to evaluate.

3. If Indeterminate form is $0\cdot\infty$ → rewrite as a quotient, bring ∞ or 0 down to denominator to create $\frac{0}{0}$ *or* $\pm\frac{\infty}{\infty}$

Example 1:

a) $\lim_{x\to\infty} e^{-x}\sqrt{x}=$

b) $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)=$

Example 2:

a) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x=$

b) $\lim_{x\to 0^+} x^x=$

c) $\lim_{x\to\infty} x^{1/x}=$

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Key

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If $\lim_{x\to a}\frac{f(x)}{g(x)}$ yields either of the indeterminate forms $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$.

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try direct substitution first.

Example 1:

a) $\lim_{x\to\infty} e^{-x}\sqrt{x} =$

$$\lim_{x\to\infty}\frac{\sqrt{x}}{e^x} = \frac{\infty}{\infty} \xrightarrow{L'H}$$

$$\lim_{x\to\infty}\frac{\frac{1}{2}x^{-1/2}}{e^x} = \frac{1}{2\sqrt{x}e^x} = \boxed{0}$$

b) $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) =$ $\infty-\infty$

$$= \frac{x-1-\ln x}{\ln x(x-1)} = \frac{0}{0}$$

$$L'H \to \lim_{x\to 1^+}\frac{1-\frac{1}{x}}{\frac{1}{x}(x-1)+\ln x(1)}\cdot\left(\frac{x}{x}\right) = \frac{x-1}{x-1+x\ln x} = \frac{0}{0}$$

$$L'H \to \lim_{x\to 1^+}\frac{1}{1+1\ln x+x(\frac{1}{x})} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

*use logs to bring exponent down.

Example 2: 1^∞

a) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x =$ $y=\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$

$$\ln y = \lim_{x\to\infty}\ln\left(1+\frac{2}{x}\right)^x$$

$$\ln y = \lim_{x\to\infty} x\ln\left(1+\frac{2}{x}\right)$$

$$\ln y = \lim_{x\to\infty}\frac{\ln(1+2x^{-1})}{x^{-1}} = \frac{0}{0}$$

$$= \lim_{x\to\infty}\frac{\frac{\cancel{-2x^{-2}}}{1+2x^{-1}}}{\cancel{-1x^{-2}}} = \frac{2}{1+\frac{2}{x}} = \frac{2}{1}$$

$$\ln y = 2$$

$$e^{\ln y} = e^2$$

$$\boxed{y=e^2}$$

b) $\lim_{x\to 0^+} x^x =$ $y=\lim_{x\to 0^+}x^x$

$$\ln y = \lim_{x\to 0^+}\ln x^x$$

$$= \lim_{x\to 0^+} x\cdot\ln x = \frac{\ln x}{x^{-1}} = \frac{-\infty}{\infty}$$

$$= \lim_{x\to 0^+}\frac{\frac{1}{x}}{-1x^{-2}} = \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -x = 0$$

$$\ln y = 0$$

$$e^{\ln y} = e^0$$

$$\boxed{y=1}$$

c) $\lim_{x\to\infty} x^{1/x} =$ $y=\lim_{x\to\infty}x^{1/x}$

$$\ln y = \lim_{x\to\infty}\ln x^{1/x}$$

$$\ln y = \lim_{x\to\infty}\frac{1}{x}\ln x$$

$$\ln y = \lim_{x\to\infty}\frac{\ln x}{x}$$

$$\ln y = \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = 0$$

$$\ln y = 0$$

$$e^{\ln y} = e^0$$

$$\boxed{y=1}$$