AP Calculus BC 9.9 Notes Geometric Power Series

We need to be able to represent certain types of rational functions as a geometric series. Rather than producing the Taylor's Rule, we will want to develop the series by manipulating a geometric series, or in some cases, using Long Division

Example 1:
First we'll do a quick review of geometric series. Geometric series are formed by multiplying by a common ratio r. Suppose I told you to start with $a_1 = 2$ and to let $r = 3$, what geometric series would you write? WHAT WOULD THE SUM BE IN EACH CASE??

What if $a_1 = 2$ and $r = -3$?

What if $a_1 = 1$ and $r = x$?

Example 2: Find the power series for $\dfrac{1}{1-x}$ centered at c = 0 by

a) Using Taylor's Rule
b) Manipulating a geometric series
c) Doing Long Division.

Find the Interval of Convergence (without using the ratio test!) Verify by graphing each:

Example 3:

Find a power series for $\frac{1}{1+x}$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Example 4:

Find a power series that represents $\frac{x}{1+x}$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Example 5:

Find a power series for $f(x)=\frac{1}{1-x^2}$, then find the interval of convergence. Find the first four nonzero terms and the general term.

Example 6:

Find a power series that represents $\frac{1}{1-2x}$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Example 7:

Find a power series for $g(x)=\frac{1}{4+x}$, centered at then find the interval of convergence. Include the first four nonzero terms and the general term.

Sometimes we cannot center our function at $x=0$. In this case, we must try to rewrite our function with the new center showing.

Example 8:

Find a power series that represents $\frac{1}{x}$ centered at $c=1$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Example 9:

Find a power series for $h(x)=\dfrac{15}{2x-1}$, centered at $c=1$, then find the interval of convergence. Include the first four nonzero terms and the general term.

We can integrate or differentiate a power series to obtain a new series. When we do this, the radius of convergence will be the same, but the interval may change (retest endpoints).

Example 10:

Find a power series that represents $\dfrac{1}{(1-x)^2}$ centered at $c=0$. Hint: what is $\int\left(\dfrac{1}{(1-x)^2}\right)dx$? What is the radius of convergence?

Example 11:

Find a power series that represents $\ln(1-x)$ centered at $c=0$. Hint: what is $\int\left(\dfrac{1}{1-x}\right)dx$? What is the radius of convergence?

Example 12:

(Similar to 2008—BC6B) Let f be the function given by $f(x)=\frac{1}{1+x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x=0$.

(b) Does the series found in part (a), when evaluated at $x=1$, converge to $f(1)$? Explain why or why not.

(c) The derivative of $\arctan x$ is $\frac{1}{1+x^2}$. Write the first four nonzero terms of the Taylor series for $\arctan x$ about $x=0$.

(d) Use the series found in part (c) to find a rational number A such that $\left|A-\arctan\left(\frac{3}{4}\right)\right|<\frac{1}{100}$. Justify your answer.