1.3 HW BC

p.67 # 6-60 even
p.68 # 67-80 all

54) $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \frac{(\sqrt{2+x}+\sqrt{2})}{(\sqrt{2+x}+\sqrt{2})} = \lim_{x\to 0} \frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})}$

$= \lim_{x\to 0} \frac{x}{x(\sqrt{2+x}+\sqrt{2})} = \frac{1}{\sqrt{2}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}}{4}}$

58) $\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x} = \lim_{x\to 0} \frac{\frac{4-(x+4)}{4(x+4)}}{x} = \lim_{x\to 0} \frac{4-x-4}{4(x+4)} \cdot \frac{1}{x}$

$4-x-4$ over $4(x+4)\cdot x$

$\lim_{x\to 0} \frac{-x}{4(x+4)} = \lim_{x\to 0} \frac{-1}{4(x+4)} = \boxed{\frac{-1}{16}}$

60) $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x\to 0} \frac{x^2+2x\Delta x+\Delta x^2-x^2}{\Delta x} = \lim_{\Delta x\to 0} \frac{\Delta x(2x+\Delta x)}{\Delta x} = \boxed{2x}$

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*Recall: 1) $\lim_{x\to 0} \frac{\sin(ax)}{(bx)} = \frac{a}{b}$ 2) $\lim_{x\to 0} \frac{1-\cos(ax)}{(ax)} = 0$

68) $\lim_{x\to 0} \frac{3(1-\cos x)}{x} = 3 \cdot \lim_{x\to 0} \frac{(1-\cos x)}{x} = 3 \cdot 0 = \boxed{0}$

70) $\lim_{\theta\to 0} \frac{\cos\theta \tan\theta}{\theta} = \lim_{\theta\to 0} \frac{\cos\theta}{\theta} \cdot \frac{\sin\theta}{\cos\theta} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \boxed{1}$

72) $\lim_{x\to 0} \frac{\tan^2 x}{x} = \lim_{x\to 0} \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{x} = \lim_{x\to 0} \boxed{\frac{\sin x}{x}}^{=1} \cdot \frac{\sin x}{\cos^2 x}$

$\lim_{x\to 0} \frac{\sin x}{\cos^2 x} = \frac{0}{1} = \boxed{0}$

74) $\lim_{\phi\to\pi} \phi \sec\phi = \pi \sec\pi = \pi(-1) = \boxed{-\pi}$

76) $\lim_{x\to \pi/4} \frac{1-\tan x}{\sin x - \cos x} = \lim_{x\to \pi/4} \frac{1-\frac{\sin x}{\cos x}}{\sin x - \cos x} = \lim_{x\to \pi/4} \frac{\frac{\cos x - \sin x}{\cos x}}{\sin x - \cos x}$ $\frac{-1}{}$

$\lim_{x\to \pi/4} \frac{-1}{\cos x} = \frac{-1}{\cos \pi/4} = \frac{-1}{\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} = \boxed{-\sqrt{2}}$

78) $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$ $\lim_{x\to 0} \left(\frac{2\sin 2x}{2x}\right)\left(\frac{3x}{3\sin 3x}\right) = 2(1)\cdot\frac{1}{3}(1)$

$= \boxed{\frac{2}{3}}$

80) $\lim_{x\to 0} \frac{\cos x - 1}{2x^2} \cdot \frac{(\cos x + 1)}{(\cos x + 1)} = \lim_{x\to 0} \frac{\cos^2 x - 1}{2x^2(\cos x + 1)} = \lim_{x\to 0} \frac{-\sin^2 x}{2x^2(\cos x + 1)}$

$= \lim_{x\to 0} -\frac{\sin x}{2x} \cdot \frac{\sin x}{x} \cdot \frac{1}{\cos x + 1} = -\frac{1}{2}\cdot 1 \cdot \frac{1}{2} = \boxed{\frac{-1}{4}}$