BC Ch. 1.4 HW p. 79 #3-72 D252

19) $\lim_{x\to\pi} \cot x \to \lim_{x\to\pi} \frac{\cos x}{\sin x} = DNE$, vertical asymptote at $x=\pi$
(nonremovable discontinuity)

20) $\lim_{x\to\pi/2} \sec x \to \lim_{x\to\pi/2} \frac{1}{\cos x} = DNE$, V.A at $x=\pi/2$

→ floor functions rounds down to next integer below.

23) $\lim_{x\to 3} \left(2 - [\![-x]\!]\right)$

$\lim_{x\to 3^-} \left(2 - [\![-2.9]\!]\right) = 2-(-3) = 5$

$\lim_{x\to 3^+} \left(2 - [\![-3.1]\!] = 2-(-4) = 6\right]$ $\lim_{x\to 3} f(x) = DNE$

24) $\lim_{x\to 1} \left(1 - \left[\!\left[\frac{-x}{2}\right]\!\right]\right)$

$\lim_{x\to 1^-} \left(1 - \left[\!\left[\frac{-0.9}{2}\right]\!\right]\right) = 1-(-1) = 2$

$\lim_{x\to 1^+} \left(1 - \left[\!\left[\frac{-1.1}{2}\right]\!\right]\right) = 1-(-1) = 2\Big]$ $\lim_{x\to 1} f(x) = 2$

32) $g(x) = \frac{1}{x^2-4}$ $[-1,2]$ $\quad g(x) = \frac{1}{(x+2)(x-2)}$ $x \neq 2, -2$, so $g(x)$ is continuous on $[-1,2)$

36) $f(x) = \cos\left(\frac{\pi x}{2}\right)$ → continuous for all values of x.

51) $f(x) = \csc(2x) = \frac{1}{\sin(2x)}$ $\sin(2x) = 0$ when $2x = \pi$, so $x = \pi/2$
Nonremovable discontinuity at integer intervals of $x = \pi/2$

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52) $f(x) = \tan\left(\frac{\pi x}{2}\right) = \frac{\sin\left(\frac{\pi x}{2}\right)}{\cos\left(\frac{\pi x}{2}\right)}$ $\cos\left(\frac{\pi x}{2}\right) = 0$ when $\frac{\pi x}{2} = \frac{\pi}{2}$, so $x = 1, 3, \ldots$
Nonremovable discontinuity at each $2k+1$ interval

Find a such that $f(x)$ is continuous

59) $f(x) = \begin{cases} 2 & x \le -1 \\ ax+b & -1 < x < 3 \\ -2 & x \ge 3 \end{cases}$

* set $2 = ax+b$ when $x=-1$

$2 = -a+b$

$\rightarrow b = 2+a$

* set $ax+b = -2$ when $x=3$

$3a+b = -2$

$\downarrow$

$3a+(2+a) = -2$

$4a = -4$

$\boxed{a=-1}$, $b = 2+a$, $\boxed{b=1}$

$f(x) = \begin{cases} 2, & x \le -1 \\ -x+1, & -1 \le x < 3 \\ -2, & x \ge 3 \end{cases}$

60) $g(x) = \begin{cases} \frac{x^2-a^2}{x-a}, & x \neq a \\ 8, & x = a \end{cases}$

* Since there is a hole at $x=a$, find $\lim_{x\to a} g(x) = 8$

$\lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} \frac{(x-a)(x+a)}{(x-a)} \Rightarrow 2a = 8$

so $\boxed{a=4}$

68) Determine x-value where $f(x)$ is not continuous

$f(x) = \begin{cases} \frac{\cos x - 1}{x}, & x < 0 \\ 5x, & x \ge 0 \end{cases}$

$\lim_{x\to 0^-} \frac{\cos x - 1}{x} = \lim_{x\to 0^-} \frac{-(1-\cos x)}{x} = 0$

$\lim_{x\to 0^+} 5x = 0$

$f(x)$ is continuous for all values of x.

a) $f(0) = 0$

b) $\lim_{x\to 0} f(x) = 0$

c) $\lim_{x\to 0} f(x) = f(0) = 0$