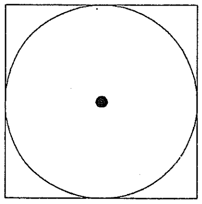
1994 AB 5-BC 2



A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$)

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

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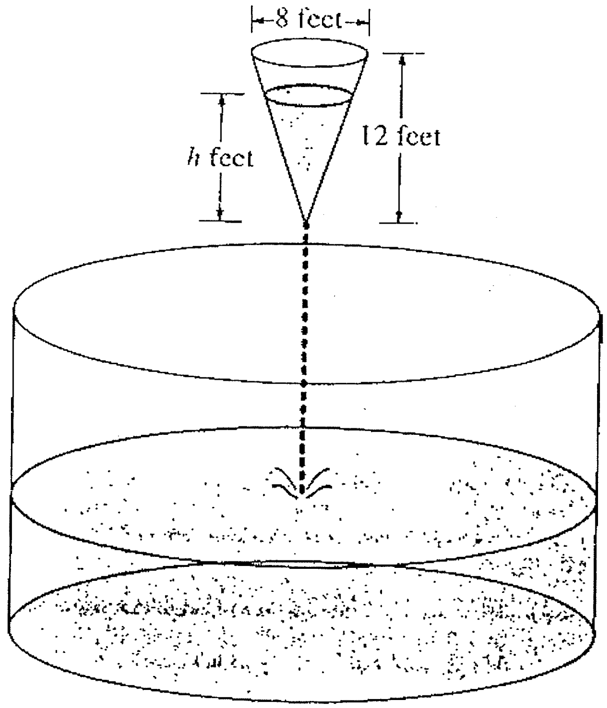


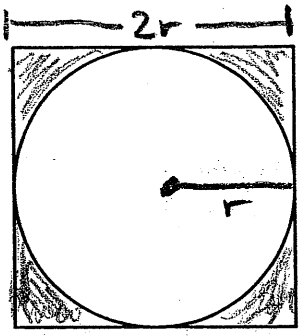
As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

Calculus 2.6: Related Rates on Free-Response Questions **Name** ____________________



1. A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency.

(a) Find the rate at which the perimeter of the square is increasing.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square.

1a) $\frac{dc}{dt} = 6$ in/s.

find $\frac{dP}{dt}$

$C = 2\pi r$

$\frac{dc}{dt} = 2\pi\left(\frac{dr}{dt}\right)$

$6 = 2\pi\left(\frac{dr}{dt}\right)$

$\boxed{\frac{dr}{dt} = \frac{3}{\pi} \text{ in/s}}$

$P = 8r$

$\frac{dp}{dt} = 8\frac{dr}{dt}$

$\frac{dp}{dt} = 8\left(\frac{3}{\pi}\right)$

$\boxed{\frac{dp}{dt} = \frac{24}{\pi} \text{ in/s}}$

$A_{SH} = A_{\square} - A_{\circ}$

$A_{SH} = (2r)^2 - \pi r^2$

$A_{SH} = 4r^2 - \pi r^2$

$\frac{dA}{dt} = 8r\frac{dr}{dt} - 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 8(5)\left(\frac{3}{\pi}\right) - 2\cancel{\pi}(5)\left(\frac{3}{\cancel{\pi}}\right)$

$\boxed{\frac{dA}{dt} = \left(\frac{120}{\pi} - 30\right) \text{in}^2/\text{s}}$

$* \; A_{\circ} = \pi r^2$

$25\cancel{\pi} = \cancel{\pi} r^2$

$\boxed{r = 5}$

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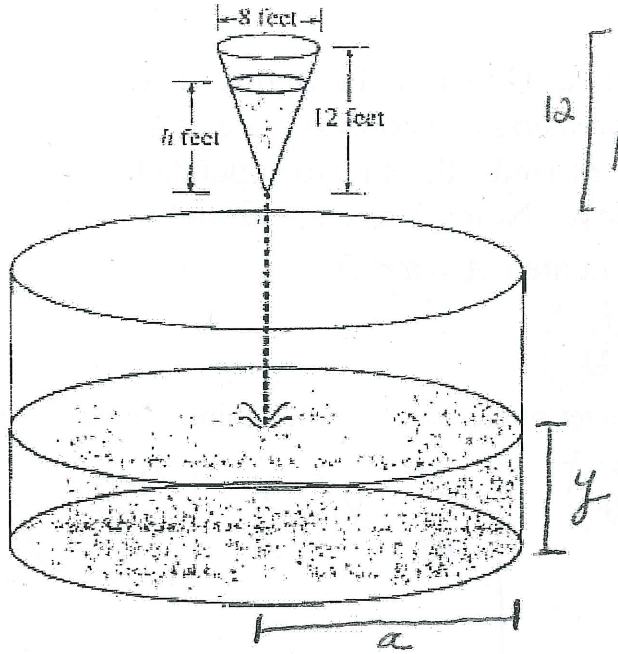

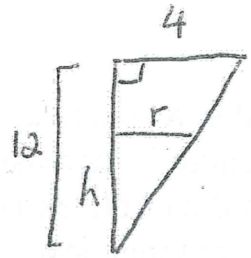

$\frac{r}{4} = \frac{h}{12}$

$12r = 4h$

$r = \frac{4}{12}h$

$\boxed{r = \frac{h}{3}}$

$\frac{dh}{dt} = h - 12$

As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^2}{9} \cdot h$

$\boxed{V = \frac{\pi}{27} h^3}$

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \cdot \left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \cdot (h-12)$

$\frac{dV}{dt} = \frac{\pi}{9}(3)^2(3-12)$

$= \frac{\pi}{\cancel{9}} \cdot \cancel{9}(-9)$

$\boxed{\frac{dV}{dt} = -9\pi \text{ ft}^3/\text{min}}$

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure. $(V = \pi a^2 y)$

$\frac{dV}{dt} = 9\pi \text{ ft}^3/\text{min}$

Area(base) $= 400\pi$

$A = \pi r^2$

$400\pi = \pi r^2$

$400 = r^2$

$20 = r$

$r = 20\text{ ft}$

$a = 20$

$V = \pi (20)^2 y$

$V = 400\pi y$

$\frac{dV}{dt} = 400\pi\left(\frac{dy}{dt}\right)$

$9\pi = 400\pi\left(\frac{dy}{dt}\right)$

$\frac{9}{400} = \frac{dy}{dt}$

$\boxed{\frac{dy}{dt} = \frac{9}{400} \text{ ft/min}}$