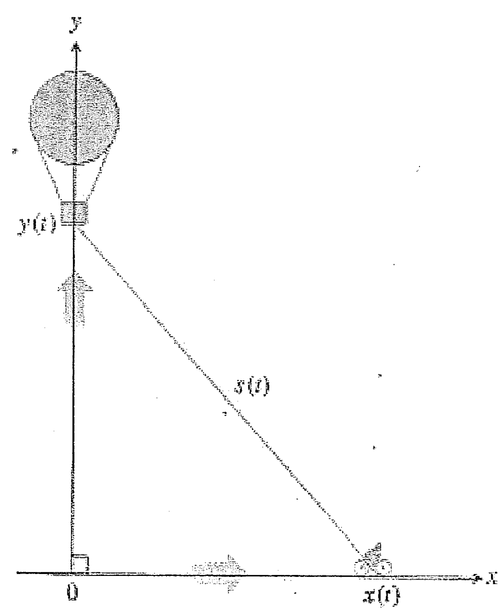
BC Calculus Related Rates WS #2

1.

A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

2.

Rising Balloon A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and balloon increasing 3 sec later (see figure)?



3. A hypothetical square shrinks at a rate of 2 m^2/min. At what rate are the diagonals of the square changing when the diagonals are 7 m each?

4. A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?

5. Suppose that a spherical balloon grows in such a way that after t seconds, $V = 4\sqrt{t}$ in^3. How fast is the radius changing after 64 seconds? ($V = \frac{4}{3}\pi r^3$)

Key

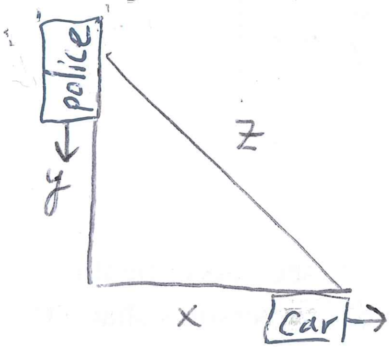
BC Calculus Related Rates WS #2

1.

A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

$$x^2+y^2=z^2$$

$$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$$



$x=0.8$ $\quad \frac{dx}{dt}=$ ___

$y=0.6$ $\quad \frac{dy}{dt}=-60$

$z=1.0$ $\quad \frac{dz}{dt}=20$

$$2(0.8)\frac{dx}{dt}+2(0.6)(-60)=2(1)(20)$$

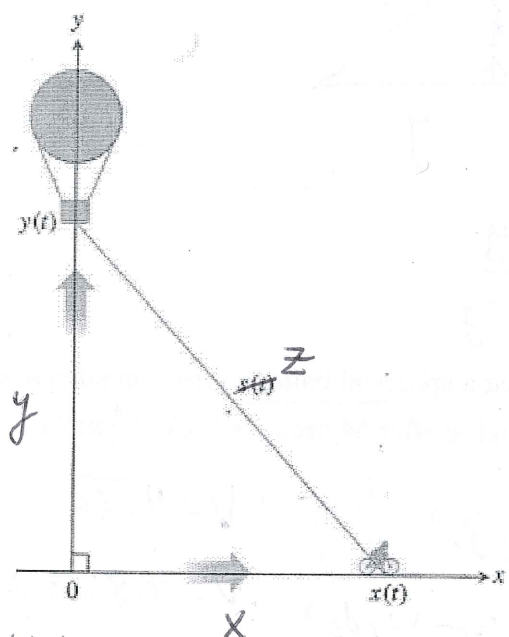
$$\frac{dx}{dt}=70\text{ mph}$$

2.

Rising Balloon A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and balloon increasing 3 sec later (see figure)?

$$x^2+y^2=z^2$$

$$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$$



$\frac{dy}{dt}=1$ ft/s

$y=68$ ft

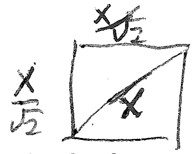
$\frac{dx}{dt}=17$ ft/s

$x=51$ ft

$z=85$

$$2(51)(17)+2(68)(1)=2(85)\left(\frac{dz}{dt}\right)$$

$$\boxed{\frac{dz}{dt}=11\text{ ft/s}}$$



3. A hypothetical square shrinks at a rate of 2 m²/min. At what rate are the diagonals of the square changing when the diagonals are 7 m each?

$A = \left(\frac{x}{\sqrt{2}}\right)^2$

$A = \frac{x^2}{2}$

$\frac{dA}{dt} = \frac{1}{2} \cdot 2x \left(\frac{dx}{dt}\right)$

$\frac{dA}{dt} = -2\ m^2/min$

$\frac{dx}{dt} = \_\_\_$

$x = 7\ m$

$-2 = 1(7)\frac{dx}{dt}$

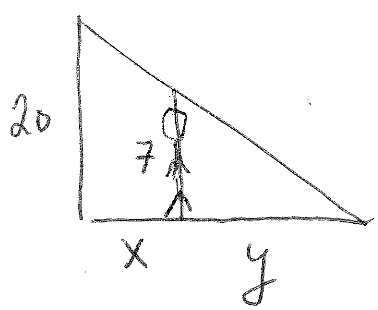
$\boxed{\frac{dx}{dt} = \frac{-2}{7}\ m/min}$

4. A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?

$\frac{dy}{dt}$ = R.O.C. of length of shadow

$\frac{dx}{dt}$ = ROC of person walking

$\frac{dx}{dt} + \frac{dy}{dt}$ = R.O.C. of tip of shadow



$\frac{7}{20} = \frac{y}{x+y}$

5. Suppose that a spherical balloon grows in such a way that after t seconds, $V = 4\sqrt{t}$ in³. How fast is the radius changing after 64 seconds? $(V = \frac{4}{3}\pi r^3)$

$\frac{dV}{dt} = 4 \cdot \frac{1}{2} t^{-1/2}$

$\frac{dV}{dt}_{64} = 2\left(\frac{1}{\sqrt{t}}\right) = 2\left(\frac{1}{\sqrt{64}}\right) = \frac{2}{8} = \frac{1}{4}$

$= 1/4\ in^3/sec$

$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3} r^2 \left(\frac{dr}{dt}\right)$

$\frac{1}{4} = 4\pi\left(\sqrt[3]{\frac{24}{\pi}}\right)^2\left(\frac{dr}{dt}\right)$

$\frac{1}{16\pi\left(\sqrt[3]{\frac{24}{\pi}}\right)^2} = \frac{dr}{dt}$

$V = 4\sqrt{64} =$

$V = 4 \cdot 8 = 32$

$V = \frac{4}{3}\pi r^3$

$32 = \frac{4\pi}{3} r^3$

$32 \cdot \frac{3}{4\pi} = r^3$

$\frac{24}{\pi} = r^3$

$r = \sqrt[3]{\frac{24}{\pi}}$