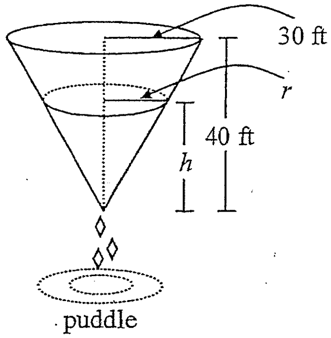
A.P. Calculus AB   Worksheet 2-6 – Related Rates WS #1

1. A conical water tank with a height of 40 ft and a radius of 30 ft is leaking at the rate of 2 ft^3 / min. When the height (h) of the water in the tank is 30 ft, at what rate is the height of the water changing? (Volume of a cone = $\frac{1}{3}\pi r^2 h$)



2. The volume of a cube is decreasing at a rate of 10 m^3 / hour. How fast is the total surface area decreasing when the surface area is 54 m^2?

3. A light is on the top of a 12 ft tall pole and a 5ft tall person is walking away from the pole at a rate of 2 ft/sec.
(a) At what rate is the tip of the shadow moving away from the pole when the person is 25 ft from the pole?
(b) At what rate is the tip of the shadow moving away from the person when the person is 25 ft from the pole?

4. A tank of water in the shape of a cone is leaking water at a constant rate of 2 ft^3/hr. The base radius of the tank is 5 ft and the height of the tank is 14 ft.

(a) At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft?

(b) At what rate is the radius of the top of the water in the tank changing when the depth of the water is 6 ft?

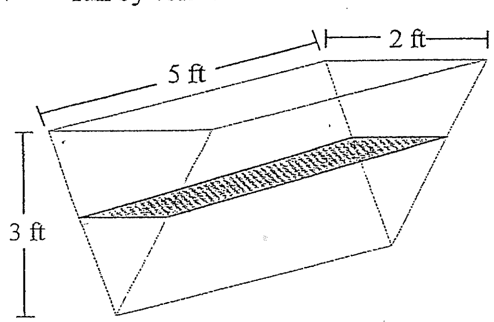
5. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)

6. The trough shown in the figure below is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at a rate of 2 cubic feet per minute. At any time, t, let h be the depth and V be the volume of water in the trough.

a) Find the volume of water in the trough when it is full.

b) What is the rate of change in h at the instant when the trough is ¼ full by volume?

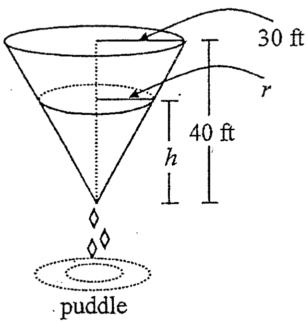
c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is ¼ full by volume?



Solution Key 2014

A.P. Calculus AB **Worksheet 2-6 – Related Rates WS #1**

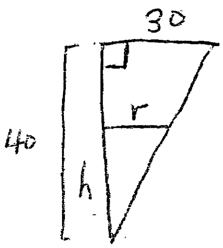
1. A conical water tank with a height of 40 ft and a radius of 30 ft is leaking at the rate of 2 ft^3 / min. When the height (h) of the water in the tank is 30 ft, at what rate is the height of the water changing? (Volume of a cone = $\frac{1}{3}\pi r^2 h$)



$\frac{dV}{dt} = -2 \text{ ft}^3/\text{min}$

$h = 30$

$\frac{dh}{dt} = \_\_\_\_$



$\frac{r}{30} = \frac{h}{40}$

$40r = 30h$

$r = \frac{3}{4}h$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{3h}{4}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{9h^2}{16} h$

$V = \frac{3\pi}{16} h^3$

$\frac{dV}{dt} = \frac{3\pi}{16} \cdot 3h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{9\pi}{16} h^2 \frac{dh}{dt}$

$-2 = \frac{9\pi}{16}(30)^2 \frac{dh}{dt}$

$\frac{-2 \cdot 16}{9\pi \cdot 30^2} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-8}{2025\pi} \text{ ft/min.}}$

2. The volume of a cube is decreasing at a rate of 10 m^3 / hour. How fast is the total surface area decreasing when the surface area is 54 m^2?

$V = x^3$

$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$

$\frac{dV}{dt} = -10 \text{ m}^3/\text{hr.}$

$S = 6x^2$

$\frac{dS}{dt} = 12x \frac{dx}{dt}$

$S = 54 m^2$

$54 = 6x^2$ $x = 3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$-10 = 3(3)^2\left(\frac{dx}{dt}\right)$

$\frac{-10}{27} = \frac{dx}{dt}$

$\frac{dS}{dt} = 12(3)\left(\frac{-10}{27}\right)$

$= \boxed{-\frac{40}{3} \text{ m}^2/\text{hr.}}$

3. A light is on the top of a 12 ft tall pole and a 5ft tall person is walking away from the pole at a rate of 2 ft/sec.

(a) At what rate is the tip of the shadow moving away from the pole when the person is 25 ft from the pole?

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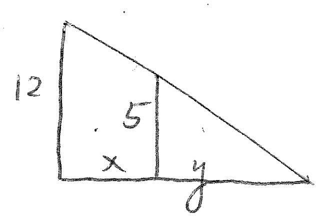
ROC for shadowlength



$\frac{5}{12} = \frac{y}{x+y}$

$5x + 5y = 12y$

$5x = 7y$

$5\frac{dx}{dt} = 7\frac{dy}{dt}$

$\frac{dx}{dt} = 2 \text{ ft/s}$

$x = 25$

$5(2) = 7\frac{dy}{dt}$

$10 = 7\frac{dy}{dt}$

$\frac{10}{7} = \frac{dy}{dt}$

a) $\frac{dx}{dt} + \frac{dy}{dt} = 10/7 + 2$

$= \frac{24}{7} \text{ ft/s}$

b) $\frac{dy}{dt} = \frac{10}{7} \text{ ft/s}$

4. A tank of water in the shape of a cone is leaking water at a constant rate of 2 ft^3/hr. The base radius of the tank is 5 ft and the height of the tank is 14 ft.

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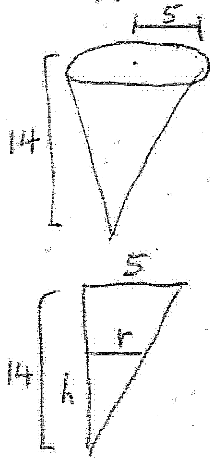


$\frac{r}{5}=\frac{h}{14}$

$\frac{dV}{dt}=-2\ ft^3/hr$

$\frac{dh}{dt}=$ ___

$h=6$ ft.

$r=\frac{5h}{14}$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{\pi}{3}\left(\frac{5h}{14}\right)^2 h$

$V=\frac{\pi}{3}\cdot\frac{25}{196}h^3$

$V=\frac{25\pi}{588}h^3$

$\frac{dV}{dt}=\frac{25\pi}{588}3h^2\frac{dh}{dt}$

$-2=\frac{75\pi}{588}(6)^2\frac{dh}{dt}$

$\frac{-2\cdot 588}{75\pi\cdot 36}=\frac{dh}{dt}$

$\boxed{\frac{dh}{dt}=\frac{-98}{225\pi}\ ft/hr.}$

Find $\frac{dr}{dt}$

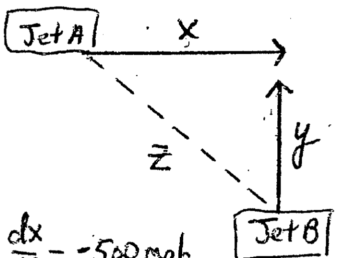
$\frac{r}{5}=\frac{h}{14}$ $\quad 14r=5h$

$14\left(\frac{dr}{dt}\right)=5\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt}=\frac{5\cdot -98}{225\pi}\cdot\frac{1}{14}$

$\boxed{\frac{dr}{dt}=\frac{-7}{45\pi}\ ft/hr.}$

5. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)



$\frac{dx}{dt}=-500$ mph

$\frac{dy}{dt}=-600$ mph

$x=100$

$z=300$

$y=282.843$ miles

$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$2(100)(-500)+2(282.843)(-600)=2(300)\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt}=-732.353\ mph}$

6. The trough shown in the figure below is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at a rate of 2 cubic feet per minute. At any time, t, let h be the depth and V be the volume of water in the trough.
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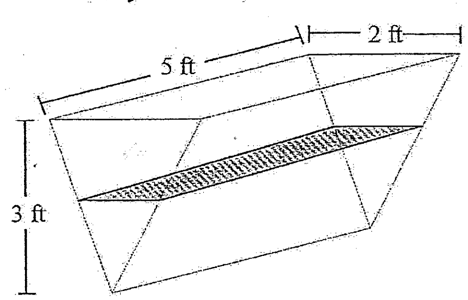

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AP Review: Related Rates

#6

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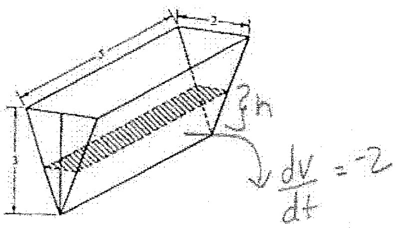

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(a) Find the volume of water in the trough when it is full.

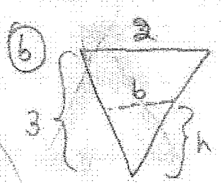
meaning volume of trough.

(b) What is the rate of change in h $\left(\frac{dh}{dt}\right)$ at the instant when the trough is $\frac{1}{4}$ full by volume?

(c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?

(a) $V = (\text{Area of } \Delta)(\text{height}) = \frac{1}{2} \cdot 2 \cdot 3 \cdot 5 = 15 \text{ ft}^3$

(b)



Similar triangles

$\frac{3}{2} = \frac{h}{b}$

$3b = 2h$

$b = \frac{2h}{3}$

When trough is $\frac{1}{4}$ full:

$V = \frac{15}{4}$

$\frac{15}{4} = \frac{5}{3}h^2$

$h = 3/2$

$V = \frac{1}{2} bh \cdot 5 = \frac{5}{2} bh$

$V = \frac{5}{\cancel{2}}\left(\frac{\cancel{2}h}{3}\right)h = \frac{5}{3}h^2$

$\frac{dV}{dt} = \frac{5}{3} \cdot 2h \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{10h}{3} \cdot \frac{dh}{dt}$

$-2 = \frac{10}{3}\left(\frac{3}{2}\right) \cdot \frac{dh}{dt}$

$\boxed{-\frac{2}{5} = \frac{dh}{dt}}$

Surface rectangle

(c) $A = 5b = 5\left(\frac{2}{3}h\right) = \frac{10}{3}h$

$\frac{dA}{dt} = \frac{10}{3} \cdot \frac{dh}{dt} = \frac{10}{3} \cdot \frac{-2}{5} = \frac{-4}{3}$