Calculus Section 7.3 Volume by Shells

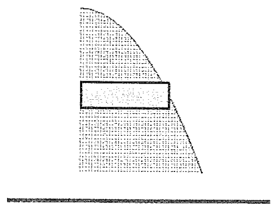
The shell method is an alternative method for finding the volume of a solid of revolution. The method is called the shell method because it uses cylindrical shells to evaluate the volume of a rotation

The Shell Method

To find the volume of a solid of revolution with the shell method, use one of the following:

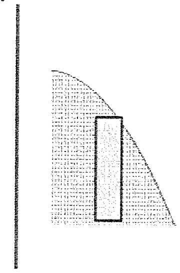
<u>Horizontal Axis of Revolution</u>

$$Volume = 2\pi\int_{c}^{d} p(y)h(y)dy$$



<u>Vertical Axis of Revolution</u>

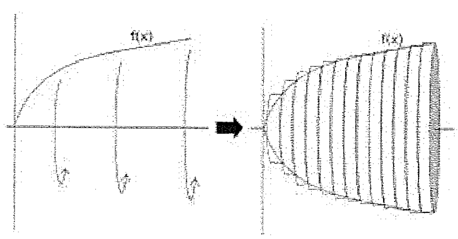
$$Volume = 2\pi\int_{a}^{b} p(x)h(x)dx$$



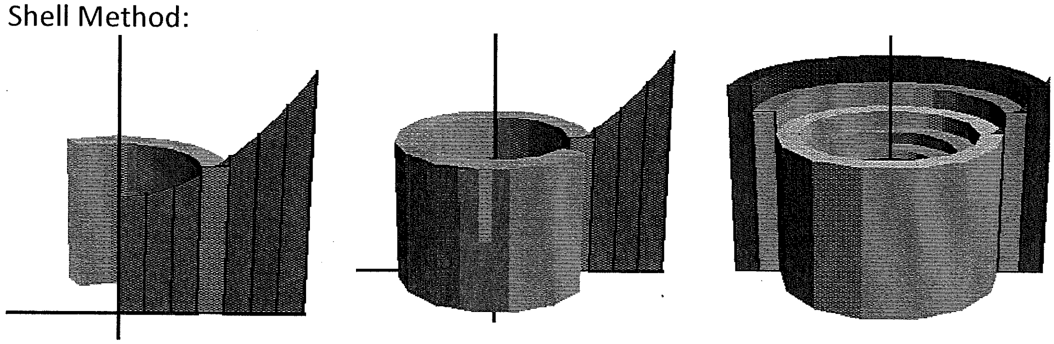
$$V = \int 2\pi\,(shell\ radius)(shell\ height)dx$$

Notice that the representative rectangles are parallel to the axis of rotation rather than perpendicular. This is opposite to the disc and washer methods.

Disc Method

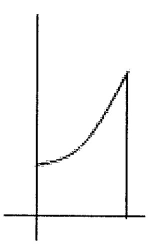


Shell Method:



Example 1

Find the volume of the solid formed by revolving $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.



Shell Method Formula: V = $\int 2\pi\ (shell\ radius)(shell\ height)dx$

Example 2: Find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$ and the x-axis $(0 \le x \le 1)$ about the y-axis.

Example 3:

Find the volume of the solid of revolution formed by revolving the region bounded by $x = e^{-y^2}$ and the y-axis $(0 \le y \le 1)$ about the x-axis.

Example 4:

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$.

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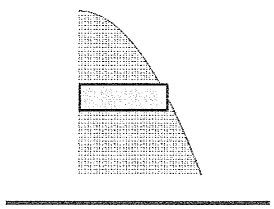
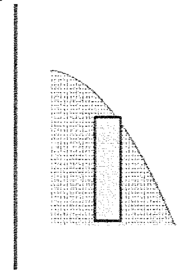


<u>Vertical Axis of Revolution</u>

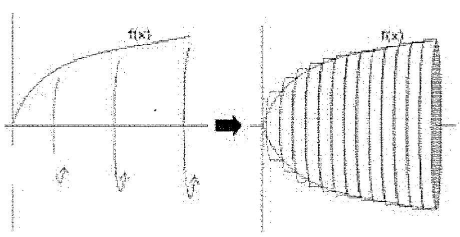
$$Volume = 2\pi\int_a^b p(x)h(x)dx$$



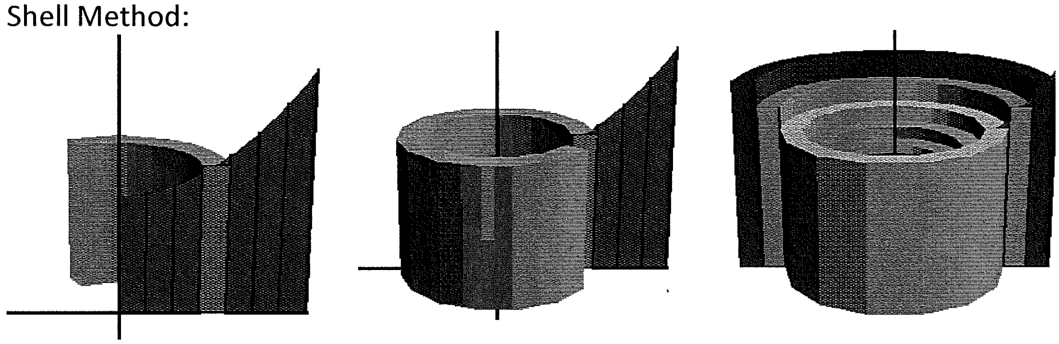
$$V = \int 2\pi\,(shell\ radius)(shell\ height)dx$$

Notice that the representative rectangles are parallel to the axis of rotation rather than perpendicular. This is opposite to the disc and washer methods.

Disc Method



Shell Method:



Example 1

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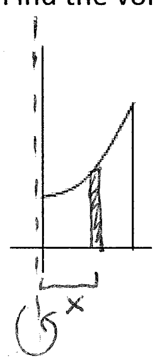


$$V = 2\pi\int_0^1 (x)(x^2+1)\,dx = 2\pi\int x^3 + x\,dx$$

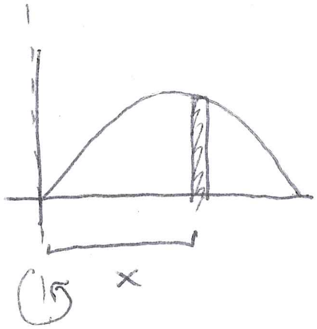
$$= 2\pi \cdot \left.\frac{x^4}{4} + \frac{x^2}{2}\right]_0^1 \quad 2\pi\left(\frac{1}{4} + \frac{1}{2} - 0\right) = 2\pi\left(\frac{1}{4} + \frac{2}{4}\right)$$

$$= 2\pi\left(\frac{3}{4}\right)$$

$$= \boxed{\frac{3\pi}{2}}$$

Shell Method Formula: $V = \int 2\pi\ (shell\ radius)(shell\ height)dx$

Example 2: Find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$ and the x-axis $(0 \le x \le 1)$ about the y-axis.



$r(x) = x$

$h(x) = x - x^3$

$$V = 2\pi \int_0^1 x(x - x^3)\,dx$$

$$2\pi \int x^2 - x^4\,dx$$

$$2\pi \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

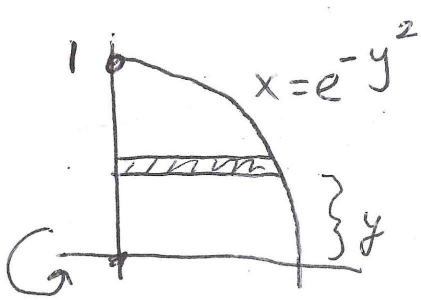
$$2\pi\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$2\pi\left(\frac{5}{15} - \frac{3}{15}\right) = 2\pi\left(\frac{2}{15}\right)$$

$$= \boxed{\frac{4\pi}{15}}$$

Example 3:

Find the volume of the solid of revolution formed by revolving the region bounded by $x = e^{-y^2}$ and the y-axis $(0 \le y \le 1)$ about the x-axis.



$$V = 2\pi \int_0^1 y \cdot e^{-y^2} dy \qquad u = -y^2 \quad \frac{du}{dy} = -2y \quad dy = \frac{du}{-2y}$$

$$2\pi \int y\, e^u \cdot \frac{du}{-2y} = -\pi \int e^u\,du$$

$$= -\pi e^{-y^2}\Big]_0^1 = -\pi\left[e^{-1} - e^0\right]$$

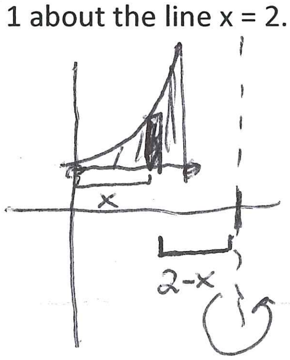
$R(y) = y$

$h(y) = e^{-y^2} - 0 = e^{-y^2}$

$$= -\pi\left(\frac{1}{e} - 1\right) = \boxed{-\frac{\pi}{e} + \pi}$$

Example 4:

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$.



$$V = 2\pi \int_0^1 (2 - x)(x^3 + x)\,dx$$

$$2\pi \int 2x^3 + 2x - x^4 - x^2\,dx$$

$$\left[\frac{2x^4}{4} + \frac{2x^2}{2} - \frac{x^5}{5} - \frac{x^3}{3}\right]_0^1$$

$R(x) = 2 - x$

$h(x) = x^3 + x + 1 - 1$

$= x^3 + x$

$$2\pi\left[\frac{1}{2} + 1 - \frac{1}{5} - \frac{1}{3}\right] = 2\pi\left(\frac{29}{30}\right) = \boxed{\frac{29\pi}{15}}$$