BC Chapter 8 Review WS #2

Solution Key

1. Find $\int \frac{1}{x(\ln x)^3}dx$ *u-sub

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\int \frac{1}{\cancel{x} \cdot u^3} \cdot \cancel{x}\,du = \int u^{-3}\,du$

$= \frac{u^{-2}}{-2} + C = \frac{-1}{2u^2} + C$

$= \boxed{\frac{-1}{2(\ln x)^2} + C}$

2. Find $\int x^3 \cos(2x)dx$ *Tab method

u	dv
$+\ x^3$	$\cos(2x)$
$-\ 3x^2$	$\frac{1}{2}\sin(2x)$
$+\ 6x$	$-\frac{1}{4}\cos(2x)$
$-\ 6$	$-\frac{1}{8}\sin(2x)$
$+\ 0$	$\frac{1}{16}\cos(2x)$

$\int \cos(2x)dx = \frac{1}{2}\sin(2x)$

$u = 2x$

$\frac{du}{dx} = 2 \quad dx = \frac{du}{2}$

$\boxed{\frac{x^3}{2}\sin 2x + \frac{3}{4}x^2\cos(2x) - \frac{3}{4}x\sin(2x) - \frac{3}{8}\cos(2x) + C}$

3. Find $\int \cot^3 2x\,dx = \int \frac{\cos^3(2x)}{\sin^3(2x)}dx$

$\int \frac{\cos^2(2x)\cdot\cos(2x)}{\sin^3(2x)}dx$

$\int \frac{[1-\sin^2(2x)]}{\sin^3(2x)}\cdot\cos(2x)dx$

$\int [\sin(2x)]^{-3} - [\sin(2x)]^{-1}\cdot\cos(2x)dx$

$u = \sin(2x)$

$\frac{du}{dx} = \cos(2x)\cdot 2$

$dx = \frac{du}{2\cos(2x)}$

$\int u^{-3} - u^{-1}\cdot\cancel{\cos(2x)}\cdot\frac{du}{2\cancel{\cos(2x)}}$

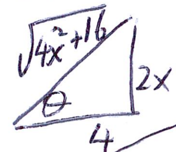
$\frac{1}{2}\left[\frac{u^{-2}}{-2}\right] - \ln|u| + C$

$\boxed{\frac{-1}{4\sin^2(2x)} - \ln|\sin(2x)| + C}$

4. Find $\int \frac{1}{x\sqrt{4x^2+16}}dx$ *$\tan\theta = \frac{u}{a}$

$\int \frac{1}{x\sqrt{(2x)^2+(4)^2}}dx$

$\tan\theta = \frac{2x}{4}$



$2x = 4\tan\theta$

$x = 2\tan\theta$

$\frac{dx}{d\theta} = 2\sec^2\theta$

$dx = 2\sec^2\theta\,d\theta$

$\sec\theta = \frac{\sqrt{4x^2+16}}{4}$

$4\sec\theta = \sqrt{4x^2+16}$

$\int \frac{2\sec^2\theta\,d\theta}{2\tan\theta\cdot 4\sec\theta}$

$= \frac{1}{4}\int \frac{\sec\theta}{\tan\theta}d\theta = \int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta}$

$\frac{1}{4}\int \csc\theta\,d\theta = \frac{-1}{4}\ln|\csc\theta + \cot\theta|$

$\boxed{\frac{-1}{4}\ln\left|\frac{\sqrt{4x^2+16}}{2x} + \frac{4}{2x}\right| + C}$

5. Find $\int \frac{x-1}{x^2(x+1)}dx = \left[\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}\right] x^2(x+1)$

$x - 1 = Ax(x+1) + B(x+1) + Cx^2$

$\boxed{x=0}$ $\ -1 = 0 + B + 0 \quad \boxed{B=-1}$

$\boxed{x=-1}$ $\ -1-1 = A(0) + B(0) + C(-1)^2 \quad \boxed{C=-2}$

$\boxed{x=1}$ $\ 1-1 = A(2) + (-1)(2) + -2(1)^2$

$0 = 2A - 2 - 2$

$4 = 2A$

$\boxed{A=2}$

$\int x^{-2} = \frac{x^{-1}}{-1}$

$\int \frac{2}{x} + \frac{-1}{x^2} + \frac{-2}{x+1}dx$

$\boxed{2\ln|x| + \frac{1}{x} - 2\ln|x+1| + C}$

6. Find $\lim_{x\to\infty} x\tan\left(\frac{1}{x}\right)$ *L'Hopital's

$\lim_{x\to\infty} \frac{\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$

$\lim_{x\to\infty} \frac{\tan(x^{-1})}{x^{-1}} = \lim_{x\to\infty} \frac{\sec^2(x^{-1})\cdot\cancel{-1x^{-2}}}{\cancel{-1x^{-2}}}$

$\lim_{x\to\infty} \sec^2\left(\frac{1}{x}\right) = \boxed{1}$

7. Find $\lim_{x\to 0^+}(e^x + x)^{2/x}$ $y = \lim_{x\to 0^+}(e^x+x)^{2x^{-1}}$ $\ln y = \lim_{x\to 0^+}(2x^{-1})\ln(e^x+x)$

$\ln y = \lim_{x\to 0^+} \frac{2\ln(e^x+x)}{x} = \frac{0}{0}$

$\ln y = \lim_{x\to 0^+} \frac{2\cdot\left(\frac{e^x+1}{e^x+x}\right)}{1} =$

$\ln y = 2\left(\frac{2}{1+0}\right) = 4$

$\ln y = 4$

$\log_e y = 4$

$e^4 = y$

$\boxed{y = e^4}$

8. $\int_0^2 \frac{1}{\sqrt[3]{x-1}}dx = \int_0^1 \frac{1}{\sqrt[3]{x-1}}dx + \int_1^2 \frac{1}{\sqrt[3]{x-1}}dx$

$u = x-1$ $\int (u)^{-1/3}du = \frac{u^{2/3}}{2/3} = \frac{3}{2}u^{2/3} = \frac{3}{2}(x-1)^{2/3}$

$\lim_{b\to 1^-} \frac{3}{2}(x-1)^{2/3}\Big]_0^b = \frac{3}{2}(b-1)^{2/3} - \frac{3}{2}(-1)^{2/3}$

$= 0 - \frac{3}{2}$

$\lim_{c\to 1^+} \frac{3}{2}(x-1)^{2/3}\Big]_c^2$

$\frac{3}{2}(1)^{2/3} - \frac{3}{2}(c-1)^{2/3}$

$-\frac{3}{2} + \frac{3}{2} = \boxed{0}$

9. $\int_1^3 \frac{2}{(x-2)^{8/3}}$

$\int_1^2 2(x-2)^{-8/3}dx + \int_2^3 2(x-2)^{-8/3}dx$

$u = x-2$ $\int u^{-8/3} = \frac{u^{-5/3}}{-5/3}$

$\lim_{b\to 2^-} -\frac{6}{5}(x-2)^{-5/3}\Big]_1^b$

$-\frac{6}{5}(0)^{-5/3} - \left[-\frac{6}{5}(1-2)^{-5/3}\right]$

$= -\frac{6}{5}$ $+$

$\lim_{c\to 2^+} -\frac{6}{5}(x-2)^{-5/3}\Big]_c^3$

$-\frac{6}{5}(3-2)^{-5/3} - \left[-\frac{6}{5}(c-2)^{5/3}\right]$

$-\frac{6}{5} + 0$

$= \boxed{\frac{-12}{5}}$

10.

Which of the following gives the value of $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$?

(A) 0 (B) 1 (C) e (D) e^2 $\boxed{\text{(E) } e^3}$

$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$

$\ln y = \lim_{x\to\infty} \ln\left[1+\frac{1}{x}\right]^{3x}$

$\ln y = \lim_{x\to\infty} (3x)\ln\left[1+\frac{1}{x}\right]$

$\ln y = \lim_{x\to\infty} \frac{3\ln\left[1+\frac{1}{x}\right]}{\frac{1}{x}} = \frac{0}{0} = \lim_{x\to\infty}\frac{3\ln\left[1+x^{-1}\right]}{x^{-1}}$ $\xrightarrow{L'H}$

$\ln y = \lim_{x\to\infty} \frac{3\cdot\left[\frac{-1\cancel{x^{-2}}}{1+x^{-1}}\right]}{-1\cancel{x^{-2}}}$

$\ln y = \lim_{x\to\infty} \frac{3}{1+\frac{1}{x}} = 3$

$\ln y = 3$

$e^3 = y$ $\boxed{y = e^3}$

11.

$\int_0^\infty x^2 e^{-x^3}\,dx =$ $u=-x^3$, $\frac{du}{dx}=-3x^2$

$\int x^2\cdot e^u\cdot\frac{du}{-3x^2} = -\frac{1}{3}\int e^u = -\frac{1}{3}e^{-x^3} = -\frac{1}{3e^{x^3}}$

(A) $-\frac{1}{3}$ (B) 0 $\boxed{(C)\ \frac{1}{3}}$ (D) 1 (E) Diverges

$\lim_{b\to\infty} -\frac{1}{3e^{x^3}}\Big]_0^b = -\frac{1}{3e^b} - \left[-\frac{1}{3e^0}\right] = 0 + \frac{1}{3} = \boxed{\frac{1}{3}}$

12.

$\int_2^\infty \frac{x}{\sqrt[3]{x^2-2}}\,dx =$ $u=x^2-2$, $\frac{du}{dx}=2x$, $dx=\frac{du}{2x}$

$\int \frac{\cancel{x}}{u^{1/3}}\cdot\frac{du}{2\cancel{x}}$, $\frac{1}{2}\int u^{-1/3}\,du$

$\frac{1}{2}\cdot\frac{u^{2/3}}{2/3} = \frac{1}{2}\cdot\frac{3}{2}u^{2/3} = \frac{3}{4}(x^2-2)^{2/3}$

(A) $\frac{3\cdot 2^{2/3}}{4}$ (B) $2^{2/3}$ (C) $-\frac{3\cdot 2^{2/3}}{4}$ (D) $-\frac{3\cdot 2^{2/3}}{2}$ $\boxed{\text{(E) Diverges}}$

$\lim_{b\to\infty} \frac{3}{4}(x^2-2)^{2/3}\Big]_2^b = \frac{3}{4}\left[b^2-2\right]^{2/3} - \frac{3}{4}\left[2^2-2\right]^{2/3} = \infty - \frac{3}{4}(2)^{2/3} = \boxed{\infty \text{ diverges}}$

*IBP/Tab method

13. $\int x^2 \sin 2x\,dx$ LIPET

$u=2x$, $\int \sin 2x\,dx = -\frac{1}{2}\cos(2x)$

u	dv
$+\ x^2$	$\sin 2x$
$-\ 2x$	$-\frac{1}{2}\cos(2x)$
$+\ 2$	$-\frac{1}{4}\sin(2x)$
$-\ 0$	$+\frac{1}{8}\cos(2x)$

$\boxed{-\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C}$

*u-sub

14. $\int \frac{x}{\sqrt[3]{4-x^2}}\,dx$ $u=4-x^2$, $\frac{du}{dx}=-2x$, $dx=\frac{du}{-2x}$

$\int \frac{x}{u^{1/3}}\cdot\frac{du}{-2x} = -\frac{1}{2}\int u^{-1/3}\,du$

$-\frac{1}{2}\cdot\frac{u^{2/3}}{2/3} = -\frac{1}{2}\cdot\frac{3}{2}u^{2/3} + C$

$= \boxed{-\frac{3}{4}(4-x^2)^{2/3} + C \quad \text{or} \quad \frac{-3\sqrt[3]{(4-x^2)^2}}{4} + C}$

15. $\int \ln\sqrt{x^2-4}\,dx$ $\quad \int \ln(x^2-4)^{1/2}\,dx$

* IBP, Long Division, Partial Fraction Decomposition

$\frac{1}{2}\int \ln(x^2-4)\,dx$ $\quad u=\ln(x^2-4)$ $\quad dv=\frac{1}{2}dx$ $\quad du=\frac{2x}{x^2-4}dx$ $\quad v=\frac{1}{2}x$

* $uv-\int v\,du$

$= \frac{1}{2}x\ln(x^2-4) - \int \frac{1}{2}x\cdot\frac{2x}{x^2-4}\,dx \leftarrow \int \frac{x^2}{x^2-4}\,dx$

$x^2-4 \overline{)\,x^2}$ $\quad 1+\frac{4}{x^2-4}$ $\quad \ominus x^2 \oplus 4$ $\quad 4$

$= \frac{1}{2}x\ln(x^2-4) - \int 1+\frac{4}{x^2-4}\,dx$

$= \frac{1}{2}x\ln(x^2-4) - \int \left(1 + \frac{A}{x+2} + \frac{B}{x-2}\right)$ $\quad A=-1$, $B=1$ $\quad x=-2$, $x=2$

$\boxed{\frac{1}{2}x\ln(x^2-4) - x + \ln|x+2| - \ln|x-2| + C}$

16. $\int \arctan 2x\,dx$ $\leftarrow$ IBP (LIPET) $\int 1\cdot\arctan(2x)\,dx$

$u=\arctan(2x)$ $\quad dv=1dx$

$du=\frac{2}{1+(2x)^2}$ $\quad v=x$

* $uv-\int v\,du$

$= x\arctan(2x) - \int x\cdot\frac{2}{1+4x^2}\,dx \leftarrow \int \frac{2x}{1+4x^2}\,dx$

$u=1+4x^2$ $\quad \frac{du}{dx}=8x$ $\quad dx=\frac{du}{8x}$ $\quad \int \frac{\cancel{2x}}{u}\cdot\frac{du}{\cancel{8x}}$

$= \boxed{x\arctan(2x) - \frac{1}{4}\ln|1+4x^2| + C}$

17. $\int \cos^3(\pi x-1)\,dx$ * Trig Integration

$\int \cos^2(\pi x-1)\cdot\cos(\pi x-1)\,dx = \int [1-\sin^2(\pi x-1)]\cdot\cos(\pi x-1)\,dx$

$u=\sin(\pi x-1)$

$\frac{du}{dx}=\cos(\pi x-1)\cdot\pi$

$dx=\frac{du}{\pi\cos(\pi x-1)}$

$\int 1-u^2\cdot\cancel{\cos(\pi x-1)}\cdot\frac{du}{\pi\cancel{\cos(\pi x-1)}}$

$\frac{1}{\pi}\int 1-u^2\,du$

$= \frac{1}{\pi}\cdot u - \frac{u^3}{3} + C$

$= \boxed{\frac{1}{\pi}\sin(\pi x-1) - \frac{1}{3}\sin^3(\pi x-1) + C}$

18. $\int \sin^2\frac{\pi x}{2}\,dx = \int \left[\sin\left(\frac{\pi x}{2}\right)\right]^2 dx$

* $\sin^2\alpha = \frac{1}{2}[1-\cos(2\alpha)]$

$\sin^2\left(\frac{\pi x}{2}\right) = \frac{1}{2}\left[1-\cos\left(2\cdot\frac{\pi x}{2}\right)\right] = \frac{1}{2}[1-\cos(\pi x)]$

$\frac{1}{2}\int [1-\cos(\pi x)]\,dx = \frac{1}{2}\int 1 - \frac{1}{2}\int \cos(\pi x)$

$u=\pi x$ $\quad \frac{du}{dx}=\pi$ $\quad dx=\frac{du}{\pi}$

$\frac{1}{2}\int [1-\cos u]\frac{du}{\pi}$

$\frac{1}{2\pi}\int 1-\cos u\,du$

$\frac{1}{2\pi}[u-\sin u]+C$

$\frac{1}{2\pi}(\pi x) - \frac{1}{2\pi}\sin(\pi x) + C$

$\boxed{\frac{x}{2} - \frac{1}{2\pi}\sin(\pi x) + C}$

19. $\int \sec^4\frac{x}{2}\,dx$ $\quad \int \left[\sec\left(\frac{x}{2}\right)\right]^2\left[\sec\left(\frac{x}{2}\right)\right]^2 dx$

* $1+\tan^2\alpha = \sec^2\alpha$

$\int \left[1+\tan^2\left(\frac{x}{2}\right)\right]\cdot\sec^2(x/2)\,dx$

$u=\tan\left(\frac{x}{2}\right)$

$\frac{du}{dx}=\sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$dx=\frac{2du}{\sec^2(x/2)}$

$\int 1+u^2\cdot\cancel{\sec^2(x/2)}\cdot\frac{2du}{\cancel{\sec^2(x/2)}}$

$2\int 1+u^2\,du = 2\left[u+\frac{u^3}{3}\right]+C$

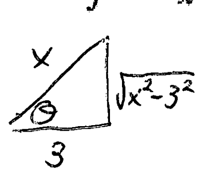
$\boxed{2\tan(x/2) + \frac{2}{3}\tan^3(x/2) + C}$

20. $\int \frac{\sqrt{x^2-9}}{x}\,dx, \quad x>3$

* Trig substitution

* $\sqrt{u^2-a^2}$, $\sec\theta=\frac{u}{a}$

$\sec\theta=\frac{x}{3}$



$x=3\sec\theta$

$\frac{dx}{d\theta}=3\sec\theta\tan\theta$

$\tan\theta=\frac{\sqrt{x^2-9}}{3}$

$\sqrt{x^2-9}=3\tan\theta$

$\theta=\sec^{-1}(x/3)$

$\int \frac{3\tan\theta}{\cancel{3\sec\theta}}\cdot\cancel{3\sec\theta}\tan\theta\,d\theta$

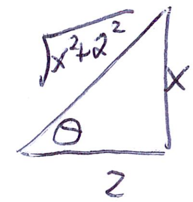
$3\int \tan^2\theta\,d\theta = 3\int [\sec^2\theta-1]\,d\theta$

$3[\tan\theta-\theta]+C$

$\boxed{3\left(\frac{\sqrt{x^2-9}}{3}\right) - 3\cdot\sec^{-1}(x/3) + C}$

21. $\displaystyle\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

$\displaystyle\int \frac{x^3}{\sqrt{2^2+x^2}}\,dx \qquad \tan\theta = \frac{u}{a} = \frac{x}{2} \qquad \tan\theta = \frac{x}{2}$



$x = 2\tan\theta$

$\frac{dx}{d\theta} = 2\sec^2\theta$

$dx = 2\sec^2\theta\, d\theta$

$\sec\theta = \frac{\sqrt{x^2+4}}{2}$

$\sqrt{x^2+4} = 2\sec\theta$

$\displaystyle\int \frac{(2\tan\theta)^3}{2\sec\theta}\cdot 2\sec^2\theta\, d\theta$

$\displaystyle\int \frac{8\tan^3\theta}{2\sec\theta}\cdot 2\sec^2\theta\, d\theta$

$8\displaystyle\int [\tan\theta]^3 \sec\theta\, d\theta$

$8\displaystyle\int [\tan^2\theta]\cdot \sec\theta\tan\theta\, d\theta$

$8\displaystyle\int [\sec^2\theta - 1]\cdot\sec\theta\tan\theta\, d\theta$

$u = \sec\theta$

$\frac{du}{d\theta} = \sec\theta\tan\theta$

$8\displaystyle\int [u^2-1]\cdot \cancel{\sec\theta\tan\theta}\cdot\frac{du}{\cancel{\sec\theta\tan\theta}}$

$8\cdot\left[\frac{u^3}{3} - u\right] + C$

$\frac{8}{3}u^3 - u = \frac{8}{3}\sec^3\theta - \sec\theta + C$

$= \frac{8}{3}\left[\frac{\sqrt{x^2+4}}{2}\right]^3 - \frac{\sqrt{x^2+4}}{2} + C$

$\boxed{\frac{1}{3}\sqrt{(x^2+4)^3} - \frac{1}{2}\sqrt{x^2+4} + C}$

22. $\displaystyle\int \frac{x-39}{x^2-x-12}\,dx$

$A=-5 \qquad B=6$

$\displaystyle\int \frac{x-39}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$

$x=4 \qquad x=-3$

$\displaystyle\int \frac{-5}{x-4} + \frac{6}{x+3}\,dx$

$= \boxed{-5\ln|x-4| + 6\ln|x+3| + C}$

$\displaystyle\int \frac{\frac{3}{2}}{x-1} + \frac{-\frac{1}{2}x+\frac{3}{2}}{x^2+1}\,dx$

$\frac{3}{2}\displaystyle\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{3-x}{x^2+1}\,dx$

$\frac{3}{2}\displaystyle\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{3}{x^2+1}\,dx - \frac{1}{2}\int \frac{x}{x^2+1}\,dx$

$\boxed{\frac{3}{2}\ln|x-1| - \frac{1}{4}\ln|x^2+1| + \frac{3}{2}\arctan(x) + C}$

23. $\left[\displaystyle\int \frac{x^2+2x}{x^3-x^2+x-1}\,dx = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}\right](x-1)(x^2+1)$

$x^2(x-1) + 1(x-1)$

$(x^2+1)(x-1)$

$x^2+2x = A(x^2+1) + (Bx+C)(x-1)$

$\boxed{x=1}$ $1^2 + 2(1) = A(2) + \cancel{(B+C)(0)}$

$3 = 2A$

$\boxed{A=\frac{3}{2}}$

$\boxed{x=0}$ $0^2+2(0) = \frac{3}{2}(1) + (B(0)+C)(-1)$

$0 = \frac{3}{2} + {}^-C$ $\boxed{C = 3/2}$

$\boxed{x=2}$ $2^2+2(2) = \frac{3}{2}(5) + \left[2B+\frac{3}{2}\right](1)$

$8 = \frac{15}{2} + \frac{3}{2} + 2B$ $\boxed{B = -1/2}$

$8 = 9 + 2B$

24. $\left[\displaystyle\int \frac{4x-2}{3(x-1)^2}\,dx = \frac{A}{x-1} + \frac{B}{(x-1)^2}\right]3(x-1)^2$

$4x-2 = 3A(x-1) + 3B$

$\boxed{x=1}$ $4-2 = 3A(0) + 3B$ $\boxed{B = 2/3}$

$\boxed{x=0}$ $-2 = 3A(-1) + 3(2/3)$

$-2 = -3A + 2$ $\quad -4 = -3A$ $\boxed{A = 4/3}$

$\displaystyle\int \frac{4/3}{x-1}\,dx + \frac{2/3}{(x-1)^2}\,dx$

$u = x-1$

$\frac{du}{dx} = 1$

$\frac{2}{3}\displaystyle\int u^{-2}\,du = \frac{2}{3}\frac{u^{-1}}{-1}$

$\boxed{\frac{4}{3}\ln|x-1| - \frac{2}{3(x-1)} + C}$

25. L'Hopital's

$$\lim_{x\to 1}\frac{(\ln x)^2}{x-1} = \frac{0}{0}$$

$$\lim_{x\to 1}\frac{2(\ln x)\left(\frac{1}{x}\right)}{1} = \lim_{x\to 1}\frac{2\ln x}{x} = \frac{2(0)}{1} = \boxed{0}$$

26. $(\infty)^0$ $\ln y = \lim_{x\to\infty}\ln\left[\ln x\right]^{2x^{-1}}$

$$\lim_{x\to\infty}(\ln x)^{2/x}$$

$$\ln y = \lim_{x\to\infty} 2x^{-1}\cdot\ln[\ln x]$$

$$\ln y = \lim_{x\to\infty}\frac{2\ln(\ln x)}{x} = \frac{\infty}{\infty}$$

$$\ln y = \lim_{x\to\infty}\frac{2\cdot\frac{\frac{1}{x}}{\ln x}}{1}$$

$$\ln y = \lim_{x\to\infty}\frac{2}{x\ln x} = 0$$

$$\ln y = 0$$

$$e^0 = y,\ \boxed{y=1}$$

27. *

$$\lim_{x\to 1^+}(x-1)^{\ln x} \qquad y = \lim_{x\to 1^+}(x-1)^{\ln x}$$

$$\ln y = \lim_{x\to 1^+}\ln(x-1)^{\ln x}$$

$$\ln y = \lim_{x\to 1^+}(\ln x)\cdot\ln(x-1)$$

$$\ln y = \lim_{x\to 1^+}\frac{\ln(x-1)}{\frac{1}{\ln x}} = \frac{\infty}{\infty}$$

$$\ln y = \lim_{x\to 1^+}\frac{\ln(x-1)}{[\ln x]^{-1}}$$

$$\ln y = \frac{\frac{1}{x-1}}{-1(\ln x)^{-2}\left(\frac{1}{x}\right)} = \frac{-[\ln x]^2}{\frac{x-1}{x}}$$

$$\lim_{x\to 1^+}\frac{-2\ln x\cdot\frac{1}{x}}{\frac{1}{x^2}} = 2x\ln x = 0$$

$$\ln y = 0\ \boxed{y=1}$$

28.

$$\lim_{x\to 1^+}\left(\frac{2}{\ln x} - \frac{2}{x-1}\right)$$

$$\lim_{x\to 1^+}\frac{2(x-1)-2\ln x}{(\ln x)(x-1)} = \lim_{x\to 1^+}\frac{2x-2-2\ln x}{(\ln x)(x-1)}$$

$$\lim_{x\to 1^+}\frac{2-0-\frac{2}{x}}{\left(\frac{1}{x}\right)(x-1)+(\ln x)(1)}\left(\frac{x}{x}\right) \quad \lim_{x\to 1^+}\frac{2x-2}{(x-1)+x\ln x} \quad \frac{0}{0}$$

$$\lim_{x\to 1^+}\frac{2}{1-0+1\ln x+x\left(\frac{1}{x}\right)} = \frac{2}{1+1} = \boxed{1}$$

Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

29.

$$\int_0^{16}\frac{1}{\sqrt[4]{x}}\,dx \qquad \lim_{b\to 0^+}\int_b^{16}x^{-1/4}\,dx = \frac{x^{3/4}}{3/4}$$

$$\lim_{b\to 0^+}\frac{4}{3}x^{3/4}\Big]_b^{16} = \lim_{b\to 0^+}\frac{4}{3}(16)^{3/4} - \frac{4}{3}(b)^{3/4} = \boxed{\frac{32}{3}}$$

30.

$$\int_0^2\frac{7}{x-2}\,dx \qquad \lim_{b\to 2^-}\int_0^b\frac{7}{x-2}\,dx$$

$$\lim_{b\to 2^-}7\ln|x-2|\Big]_0^b = 7\ln|b-2| - (7\ln|2|)\ \boxed{\infty\ \text{Diverges}}$$

31.

$$\int_1^\infty x^2\ln x\,dx \qquad \lim_{b\to\infty}\int_1^b x^2\ln x\,dx$$

IBP

$u=\ln x$ $dv=x^2$

$du=\frac{1}{x}$ $v=\frac{x^3}{3}$

$$(\ln x)\left(\frac{x^3}{3}\right) - \int\frac{1}{x}\cdot\frac{x^3}{3}\,dx$$

$$\frac{1}{3}\int x^2\,dx$$

$$\lim_{b\to\infty}\ln(x)\cdot\frac{x^3}{3} - \frac{x^3}{9}\Big]_1^b \ \boxed{=\infty\ \text{Diverges}}$$

32. *IBP

$$\int_1^\infty\frac{\ln x}{x^2}\,dx \qquad \lim_{b\to\infty}\int_1^b(\ln x)x^{-2}\,dx$$

$u=\ln x$ $dv=x^{-2}$

$\frac{du}{dx}=\frac{1}{x}$ $v=\frac{x^{-1}}{-1}$

$$-\frac{\ln x}{x} - \int\frac{-1}{x}\left(\frac{1}{x}\right)dx$$

$$+\int x^{-2}\,dx$$

$$\lim_{b\to\infty} -\frac{\ln x}{x} + \frac{x^{-1}}{-1}$$

$$\lim_{b\to\infty} -\frac{\ln x}{x} - \frac{1}{x}\Big]_1^b$$

$$-\frac{\ln b}{b} - \frac{1}{b} - \left[-\frac{\ln 1}{1} - \frac{1}{1}\right]$$

$$0 + 1 = \boxed{1}$$

33.

$$\int_2^\infty\frac{1}{x\sqrt{x^2-4}}\,dx = \int_2^3\frac{1}{x\sqrt{x^2-4}}\,dx + \int_3^\infty\frac{1}{x\sqrt{x^2-4}}\,dx$$

$$\lim_{b\to 2^+}\frac{1}{2}\operatorname{arcsec}\frac{|x|}{2}\Big]_b^3 \qquad \lim_{c\to\infty}\frac{1}{2}\operatorname{arcsec}\frac{|x|}{2}\Big]_3^c$$

$$\lim_{b\to 2^+}\frac{1}{2}\operatorname{arcsec}\frac{|3|}{2} - \frac{1}{2}\operatorname{arcsec}\left|\frac{b}{2}\right| + \lim_{c\to\infty}\frac{1}{2}\operatorname{arcsec}\left(\frac{c}{2}\right) - \frac{1}{2}\operatorname{arcsec}\left(\frac{3}{2}\right)$$

$$\frac{1}{2}\operatorname{arcsec}\frac{3}{2} - \frac{1}{2}(0) + \frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{2}\operatorname{arcsec}\left(\frac{3}{2}\right)$$

$$= 0 - 0 + \frac{\pi}{4} - 0$$

$$= \boxed{\frac{\pi}{4}}$$