

BC Calculus Chapter 4.1-4.2 Practice Problems

1. Approximate the area represented by $\int_{-1}^5 x^2 - 4x - 5 dx$

a. Find Lower Sum using 4 equal intervals

b. Find Upper Sum using 4 equal intervals

c. Find Midpoint Sum using 5 equal intervals

d. Find Trapezoid Approximation using 5 equal intervals

2. Given $g'(x) = 12x + 6$ and $g(0) = 4$ and $g(1) = -2$. Find $g(x)$.

Summation Rules:

Limit Definition of a Definite Integral

1. $\sum_{i=1}^n c =$

2. $\sum_{i=1}^n i =$

3. $\sum_{i=1}^n i^2 =$

4. $\sum_{i=1}^n i^3 =$

3. Find $\sum_{i=1}^{15} (i-3)^2$

4. Use Limit Definition of an Integral to find the definite integral for $f(x) = 1 - x^2$ in interval $[-3, 1]$

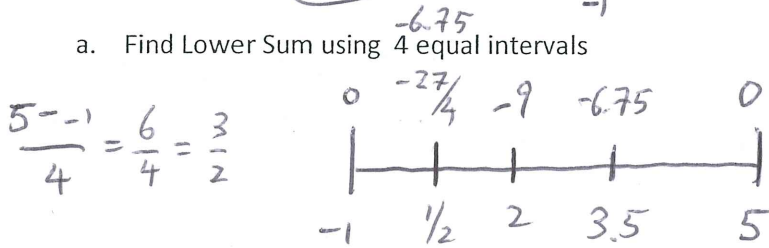
BC Calculus Chapter 4.1-4.2 Practice Problems

Key

1. Approximate the area represented by $\int_{-1}^5 x^2 - 4x - 5 dx$

width = $\frac{b-a}{n}$

a. Find Lower Sum using 4 equal intervals

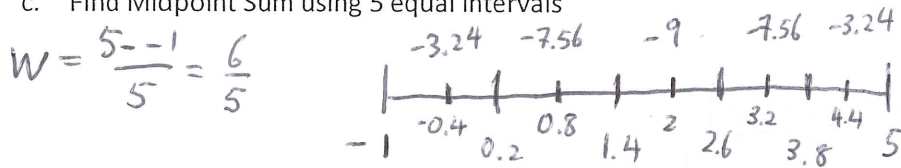


$$\int_{-1}^5 f(x) dx \approx \frac{3}{2} [f(-1) + f(1/2) + f(3.5) + f(5)] = \boxed{20.25}$$

b. Find Upper Sum using 4 equal intervals

$$\frac{3}{2} [f(1/2) + f(2) + f(2) + f(3.5)] = \frac{3}{2} [6.75 + 9 + 9 + 6.75] = \boxed{47.25}$$

c. Find Midpoint Sum using 5 equal intervals



$$\frac{6}{5} [f(-0.4) + f(0.8) + f(2) + f(3.2) + f(4.4)] = \boxed{36.72}$$

d. Find Trapezoid Approximation using 5 equal intervals

$$\frac{W}{2} [h_1 + 2h_2 + 2h_3 + \dots + h_n] \quad A = \frac{W}{2} [h_1 + h_2]$$

$$\frac{6}{5} \cdot \frac{1}{2} [f(-1) + 2f(0.2) + 2f(1.4) + 2f(2.6) + 2f(3.8) + f(5)] = \boxed{34.56}$$

2. Given $g''(x) = 12x + 6$ and $g(0) = 4$ and $g(1) = -2$. Find $g(x)$.

$$g'(x) = \frac{12x^2}{2} + 6x + C_1$$

$$g'(x) = 6x^2 + 6x + C_1$$

$$g(x) = \frac{6x^3}{3} + \frac{6x^2}{2} + C_1x + C_2$$

$$g(x) = 2x^3 + 3x^2 + C_1x + C_2$$

$$4 = 0 + 0 + 0 + C_2 \quad \underline{\underline{C_2 = 4}}$$

$$g(x) = 2x^3 + 3x^2 + C_1x + 4$$

$$-2 = 2(1)^3 + 3(1)^2 + C_1 + 4$$

$$-2 = 2 + 3 + 4 + C_1$$

$$\underline{\underline{-11 = C_1}}$$

$$\boxed{g(x) = 2x^3 + 3x^2 - 11x + 4}$$

Summation Rules:

$$1. \sum_{i=1}^n c = cn$$

$$2. \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$3. \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$4. \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

Limit Definition of a Definite Integral

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n \text{width} \cdot f[\text{left} + \text{width} \cdot i]$$

$$\frac{b-a}{n} \cdot f\left[a + \frac{b-a}{n} i\right]$$

3. Find $\sum_{i=1}^{15} (i-3)^2$

$$\sum_{i=1}^{15} i^2 - 6i + 9$$

$$\sum i^2 - 6 \sum i + 9 \sum 1$$

$$\frac{n(n+1)(2n+1)}{6} - 6 \cdot \frac{n(n+1)}{2} + 9 \cdot n$$

$$\frac{15(16)(31)}{6} - 6 \cdot \frac{15(16)}{2} + 9 \cdot 15 = \boxed{655}$$

4. Use Limit Definition of an Integral to find the definite integral for $f(x) = 1 - x^2$ in interval $[-3, 1]$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{4}{n} \cdot f\left[-3 + \frac{4}{n} i\right]$$

$$\frac{4}{n} \left[1 - \left(-3 + \frac{4}{n} i\right)^2 \right]$$

$$\frac{4}{n} \left[1 - \left(\frac{16}{n^2} i^2 - \frac{24}{n} i + 9 \right) \right]$$

$$\frac{4}{n} \left[-8 - \frac{16}{n^2} i^2 + \frac{24}{n} i \right]$$

$$= \frac{-32}{n} (n) - \frac{64}{n^3} \left[\frac{n(n+1)(2n+1)}{6} \right] + \frac{96}{n^2} \left[\frac{n(n+1)}{2} \right]$$

$$\frac{-32n}{n} - \frac{64}{n^3} \left[\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \right] + \frac{96}{n^2} \left[\frac{n^2}{2} + \frac{n}{2} \right]$$

$$-32 - \frac{64}{3} + \frac{96}{2} = \boxed{\frac{-16}{3}}$$