BC Calculus Chapter 4.1-4.2 Practice Problems

1. Approximate the area represented by $\int_{-1}^{5} x^2 - 4x - 5dx$

a. Find Lower Sum using 4 equal intervals

b. Find Upper Sum using 4 equal intervals

c. Find Midpoint Sum using 5 equal intervals

d. Find Trapezoid Approximation using 5 equal intervals

2. Given g ″(x) = 12x + 6 and g(0) = 4 and g(1) = -2. Find g(x).

Summation Rules:

Limit Definition of a Definite Integral

1. $\sum_{i=1}^{n} c =$
2. $\sum_{i=1}^{n} i =$
3. $\sum_{i=1}^{n} i^2 =$
4. $\sum_{i=1}^{n} i^3 =$

3. Find $\sum_{i=1}^{15} (i-3)^2$

4. Use Limit Definition of an Integral to find the definite integral for $f(x) = 1 - x^2$ in interval [-3, 1]

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Key

width $= \frac{b-a}{n}$

1. Approximate the area represented by $\int_{-1}^{5} x^2 - 4x - 5dx$

a. Find Lower Sum using 4 equal intervals

-6.75

$\frac{5--1}{4} = \frac{6}{4} = \frac{3}{2}$

0 $\quad -27/4 \quad -9 \quad -6.75 \quad 0$

$-1 \quad 1/2 \quad 2 \quad 3.5 \quad 5$

$$\int_{-1}^{5} f(x)\,dx \approx \frac{3}{2}\left[f(-1) + f(1/2) + f(3.5) + f(5)\right] = \boxed{20.25}$$

b. Find Upper Sum using 4 equal intervals

$$\frac{3}{2}\left[f(1/2) + f(2) + f(2) + f(3.5)\right] = \frac{3}{2}\left[6.75 + 9 + 9 + 6.75\right] = \boxed{47.25}$$

c. Find Midpoint Sum using 5 equal intervals

$W = \frac{5--1}{5} = \frac{6}{5}$

$-3.24 \quad -7.56 \quad -9 \quad -7.56 \quad -3.24$

$-1 \quad -0.4 \quad 0.2 \quad 0.8 \quad 1.4 \quad 2 \quad 2.6 \quad 3.2 \quad 3.8 \quad 4.4 \quad 5$

$$\frac{6}{5}\left[f(-0.4) + f(0.8) + f(2) + f(3.2) + f(4.4)\right] = \boxed{36.72}$$

d. Find Trapezoid Approximation using 5 equal intervals

$$\frac{W}{2}\left[h_1 + 2h_2 + 2h_3 + \dots + h_n\right] \qquad A = \frac{W}{2}\left[h_1 + h_2\right]$$

$$\frac{6}{5} \cdot \frac{1}{2}\left[f(-1) + 2f(0.2) + 2f(1.4) + 2f(2.6) + 2f(3.8) + f(5)\right] = \boxed{34.56}$$

2. Given g ''(x) = 12x + 6 and g(0) = 4 and g(1) = -2. Find g(x).

$g'(x) = \frac{12x^2}{2} + 6x + C_1$

$g'(x) = 6x^2 + 6x + C_1$

$g(x) = \frac{6x^3}{3} + \frac{6x^2}{2} + C_1x + C_2$

$g(x) = 2x^3 + 3x^2 + C_1x + C_2$

$4 = 0 + 0 + 0 + C_2 \qquad C_2 = 4$

$g(x) = 2x^3 + 3x^2 + C_1x + 4$

$-2 = 2(1)^3 + 3(1)^2 + C_1 + 4$

$-2 = 2 + 3 + 4 + C_1$

$-11 = C_1$

$\boxed{g(x) = 2x^3 + 3x^2 - 11x + 4}$

Summation Rules:

1. $\sum_{i=1}^{n} c = cn$
2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Limit Definition of a Definite Integral

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \text{width} \cdot f[\text{left} + \text{width}\cdot i]$$

$$\frac{b-a}{n} \cdot f\left[a + \frac{b-a}{n} i\right]$$

3. Find $\sum_{i=1}^{15} (i-3)^2$

$$\sum_{i=1}^{15} i^2 - 6i + 9$$

$$\sum i^2 - 6\sum i + 9\sum 1$$

$$\frac{n(n+1)(2n+1)}{6} - 6\cdot\frac{n(n+1)}{2} + 9\cdot n$$

$$\frac{15(16)(31)}{6} - 6\cdot\frac{15(16)}{2} + 9\cdot 15 = \boxed{655}$$

4. Use Limit Definition of an Integral to find the definite integral for f(x) = 1 – x^2 in interval [-3, 1]

$$\lim_{n\to\infty} \sum_{n}^{n} \frac{4}{n} \cdot f\left[-3 + \frac{4}{n} i\right]$$

$$\frac{4}{n}\left[1 - \left(-3 + \frac{4}{n}i\right)^2\right]$$

$$\frac{4}{n}\left[1 - \left(\frac{16}{n^2}i^2 - \frac{24}{n}i + 9\right)\right]$$

$$\frac{4}{n}\left[-8 - \frac{16}{n^2}i^2 + \frac{24}{n}i\right]$$

$$\frac{-32}{n}(n) - \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{96}{n^2}\left[\frac{n(n+1)}{2}\right]$$

$$\frac{-32n}{n} - \frac{64}{n^3}\left[\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right] + \frac{96}{n^2}\left[\frac{n^2}{2} + \frac{n}{2}\right]$$

$$-32 - \frac{64}{3} + \frac{96}{2} = \boxed{-\frac{16}{3}}$$