BC Ch. 7 Chapter Review #1 : No calculators

Disc Method: $V = \pi \int_{x_1}^{x_2} [R(x)]^2 \, dx$

Washer Method $V = \pi \int_a^b (R(x)^2 - r(x)^2) dx$

Volumes cross sections: $V = \int_a^b (\textit{Area} \text{ of cross section})(dx \, or \, dy)$

Shell method: $V = 2\pi \int_a^b (\text{shell radius})(\text{shell height}) dx$

(derived from lateral surface area of cylinder: $2\pi r h$)

arc length $s = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$

Area of Surface of Revolution:

Derived from frustum surface area: $S = 2\pi r l$ (l is slant height)

Horizontal AOR : $S = 2\pi \int_a^b [f(x)] \sqrt{1 + [f'(x)]^2} \, dx$

Vertical AOR : $S = 2\pi \int_a^b x \sqrt{1 + [f'(x)]^2} \, dx$

1. Find the area(s) bounded by $y = -\sin x$ and $y = \cos x$, $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

2. Find the volume of the solid generated by revolving the region bounded by $x = 4 - y^2$, $x = 1$, and the x-axis …

   a) About the line $x = 1$

   b) About the line $x = 4$

2. Find the volume of the solid generated by revolving the region bounded by $x = 4 - y^2$, $x = 1$, and the x-axis ...
   c) About the line $x = -1$ (Using Shell Method)

   d) The region represents the base of a solid. The cross-sections of the solid are right isosceles triangles perpendicular to the x-axis
      a. With the leg on the base of the solid

      b. With the hypotenuse on the base of the solid

      c. For this solid, each cross section perpendicular to the x-axis is a trapezoid of height 5 with its upper base twice the length of its lower base lying on the region. (Trapezoid Area $= \frac{h}{2}(b_1 + b_2)$)

BC Ch. 7 Chapter Review #1 : No calculators

Key

Disc Method: $V = \pi \int_{x_1}^{x_2} [R(x)]^2\,dx$

Washer Method $V = \pi \int_a^b (R(x)^2 - r(x)^2)\,dx$

Volumes cross sections: $V = \int_a^b (\textit{Area}\text{ of cross section})(dx \text{ or } dy)$

Shell method: $V = 2\pi \int_a^b (\text{shell radius})(\text{shell height})\,dx$

(derived from lateral surface area of cylinder: $2\pi r h$)

arc length $s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

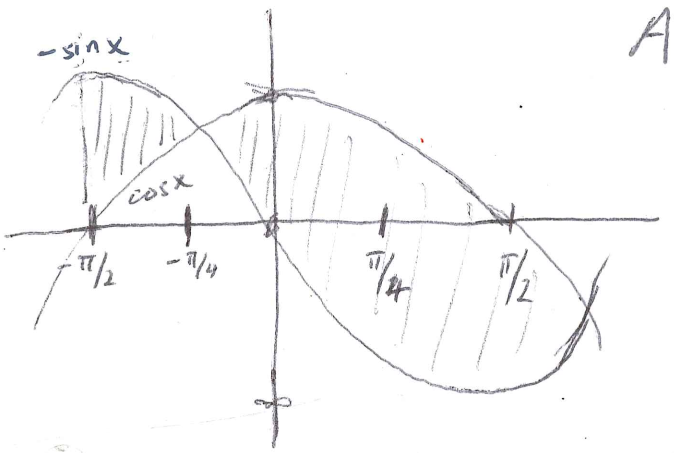
Area of Surface of Revolution:

Derived from frustum surface area: $S = 2\pi r l$ (l is slant height)

Horizontal AOR : $S = 2\pi \int_a^b [f(x)]\sqrt{1+[f'(x)]^2}\,dx$

Vertical AOR : $S = 2\pi \int_a^b x\sqrt{1+[f'(x)]^2}\,dx$

1. Find the area(s) bounded by $y = -\sin x$ and $y = \cos x$, $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$



$$A = \int_{-\pi/2}^{-\pi/4} -\sin x - \cos x\,dx + \int_{-\pi/4}^{\pi/2} \cos x - (-\sin x)\,dx$$

$-\sin x = \cos x$

$\frac{\sin x}{\cos x} = -1$

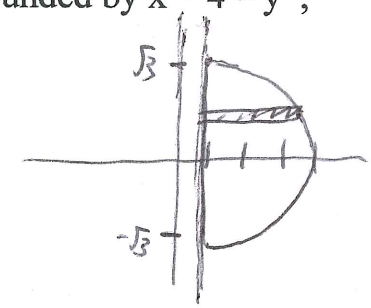
$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4} \to -\frac{\pi}{4}$

$y = \sqrt{4-x}$

2. Find the volume of the solid generated by revolving the region bounded by $x = 4 - y^2$, $x = 1$, and the x-axis …

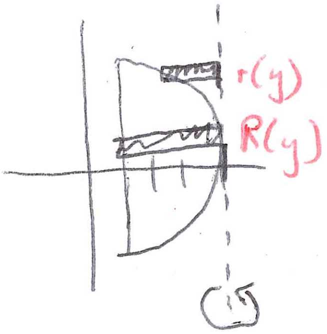
$4 - y^2 = 1 \quad y^2 = 3 \quad y = \pm\sqrt{3}$



a) About the line x = 1 (Disc)

$$V = \pi \int_{-\sqrt{3}}^{\sqrt{3}} (4 - y^2 - 1)^2\,dy \to V = \pi \int_{-\sqrt{3}}^{\sqrt{3}} (3 - y^2)^2\,dy$$

b) About the line x = 4 (Washer)



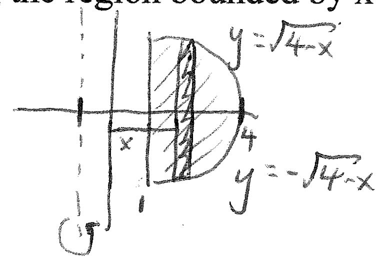
$$V = \pi \int_{-\sqrt{3}}^{\sqrt{3}} 3^2 - y^4\,dy$$

$R(y) = 4 - 1 = 3$

$r(y) = 4 - (4 - y^2) = y^2$

2. Find the volume of the solid generated by revolving the region bounded by $x = 4 - y^2$, $x = 1$, and the x-axis …

c) About the line $x = -1$ (Using Shell Method)



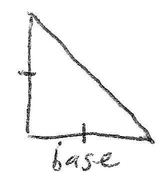
$$V = 2\pi \int_1^4 (1+x)\left(2\sqrt{4-x}\right) dx$$

$$h(x) = \sqrt{4-x} - \left(-\sqrt{4-x}\right)$$

d) The region represents the base of a solid. The cross-sections of the solid are right isosceles triangles perpendicular to the x-axis

base $= 2\sqrt{4-x}$

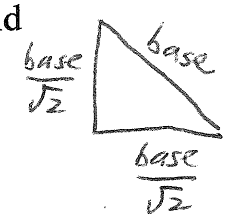
a. With the leg on the base of the solid



$$V = \int_1^4 \frac{1}{2}(\text{base})^2\, dx$$

$$V = \int_1^4 \frac{1}{2}\left[2\sqrt{4-x}\right]^2 dx$$

b. With the hypotenuse on the base of the solid



$$V = \int_1^4 \frac{1}{2}\left[\frac{\text{base}}{\sqrt{2}}\right]^2 dx$$

$$V = \frac{1}{4}\int_1^4 \left[2\sqrt{4-x}\right]^2 dx$$

c) Trapezoid with one base on the the solid. The other base is twice the length of side on the solid height of 5.

$A_T = \frac{h}{2}[b_1 + b_2]$

$$V = \frac{5}{2}\int_1^4 2\sqrt{4-x} + 4\sqrt{4-x}\, dx$$

7.4 Review

1) Find arc length of graph over the indicated interval

$$y = \frac{3}{2}x^{2/3} + 4 \quad [1, 27]$$

$$S = \int_1^{27} \sqrt{1 + \left(\frac{1}{x^{1/3}}\right)^2} = \int_1^{27} \sqrt{1 + \frac{1}{x^{2/3}}} = \int \sqrt{\frac{x^{2/3}+1}{x^{2/3}}}$$

$$= \int_1^{27} \frac{1}{x^{1/3}} \sqrt{x^{2/3}+1}\, dx$$

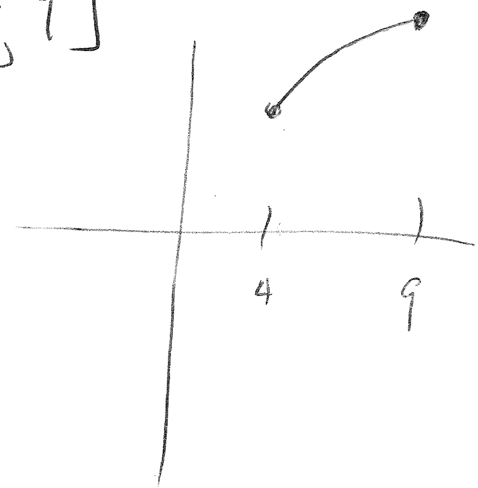
$u = x^{2/3} + 1$

$\frac{du}{dx} = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} \quad dx = \frac{3x^{1/3}}{2} du$

$$= \frac{3}{2}\int \frac{1}{\cancel{x^{1/3}}} u^{1/2} \cdot \cancel{x^{1/3}}\, du \quad \frac{3}{2} \frac{u^{3/2}}{3/2} = \frac{3}{2} \cdot \frac{2}{3} u^{3/2}$$

$$\left[\left[x^{2/3}+1\right]^{3/2}\right]_1^{27} = \boxed{10^{3/2} - 2^{3/2}}$$

2) Find Area of surface by revolving curve about the given axis: $y = 2\sqrt{x}$ $[4, 9]$



a) x-axis

b) y-axis

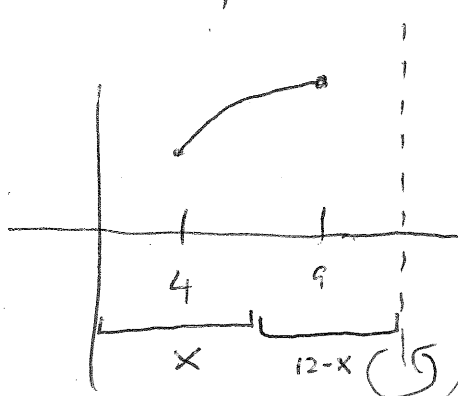
c) $x = 12$

d) $y = 2$

a) $$S = 2\pi \int_4^9 2\sqrt{x}\,\sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2}\,dx$$

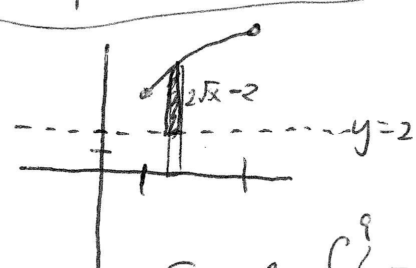
b) $$S = 2\pi \int_4^9 x\,\sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2}\,dx$$

c)



$$S = 2\pi \int_4^9 (12 - x)\sqrt{1 + \frac{1}{x}}\,dx$$

d)



$$S = 2\pi \int_4^9 (2\sqrt{x} - 2)\sqrt{1 + \frac{1}{x}}\,dx$$