Name _____

AP Calculus BC   Pre-Final Review Project

Due Date: Beginning of class, Monday, November 28th, 2016

- ALL WORK MUST BE DONE INDIVIDUALLY. YOU MAY <u>NOT</u> WORK TOGETHER. HOWEVER, YOU MAY REFER TO YOUR NOTES, HOMEWORK, OR THE TEXTBOOK AS NEEDED FOR ASSISTANCE.
- YOU MUST SIGN THE HONOR CODE STATEMENT AT THE END OF THE DOCUMENT TO RECEIVE CREDIT FOR THE PROJECT.
- FULL WORK MUST BE SHOWN FOR EVERY PROBLEM TO RECEIVE CREDIT.
- PROBLEMS SHOULD BE SOLVED WITHOUT USING A CALCULATOR UNLESS OTHERWISE DIRECTED

Quiz   140

Grading: This project will count as a project grade of 100 points. You will be graded 50% on completion and 50% on accuracy. For the accuracy portion, 8 short answer problems will be selected at random and graded out of 5 points each, one long answer problem will be selected and graded out of 10 points.
Problems with no work shown will not be counted for completion.

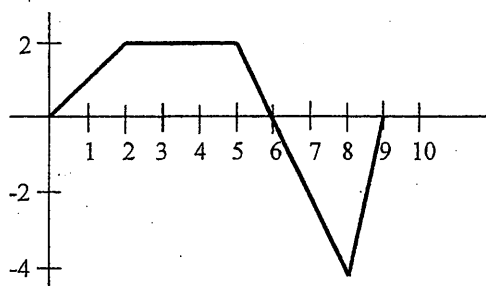|	
1. Find $\lim\limits_{x\to\infty}\dfrac{3x^2}{x^2-4x+7}$	2. Find the derivative of $f(x)=\dfrac{3x^2}{x-7}$
3. The velocity function of an object is $v(t)=6t^2-8t$, where v is in m/s. What is the average velocity, in m/s, over the time interval $[2, 5]$?	4. Evaluate $\displaystyle\int \dfrac{1}{x^2 e^{1/x}}\,dx$

5. If $f(x) = xe^x$, find $f''(x)$	6. Find the equation of the line tangent to the curve $3x^2 - xy + y^2 = 5$ at the point (1, 2)

7. This graph shows the velocity of an object moving along the x-axis for $0 \le t \le 9$. Given that at $t = 8$ the object was at position $x = 10$, what was the position at $t = 5$?	8. If it takes the amount of money you have invested in an account 20 years to double in size, what is the annual percentage rate if the interest is compounded continuously?

9. Evaluate $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{x - 4}$	10. Find the value of $f'\left(\dfrac{\pi}{6}\right)$ if $f(x) = \sin x$

11. Find the function with the derivative represented by $\lim\limits_{\Delta x \to 0} \dfrac{((x + \Delta x)^3 + 1) - (x^3 + 1)}{\Delta x}$

12. Evaluate the following integral: $\int_1^3 x^2 \, dx$

13. Which of the following is **not** a requirement for the continuity of a function at some value c? Justify your answer.

a) $f(c)$ exists

b) $\lim\limits_{x \to c^+} f(x) = \lim\limits_{x \to c^-} f(x)$

c) $\lim\limits_{x \to c} f(x) = f(c)$

d) $\lim\limits_{x \to c} f(x)$ exists

14. What can you conclude from the following information: $f(6) = 7, f'(6) = 0$, $f''(6) = -3$? (Justify your answer

a) $(6, 7)$ is a relative maximum value
b) $(6, 7)$ is a relative minimum value
c) $(6, 0)$ is a root of the function
d) There is not enough information to determine anything
e) None of the above

15. Find $\dfrac{dy}{dx}$ for if: $y = 2\sin x - 4\cos x + x$

16. Evaluate the following integral: $\int 3x \sin(x^2) \, dx$

17. Evaluate $\displaystyle\lim_{x\to 5}\frac{3-\sqrt{2x-1}}{x-5}$

18. Find $\dfrac{dy}{dx}$ for the following: $y^2 - 3xy + 7x = 2$

19. _____ Choose the correct statement given that $\displaystyle\int_0^7 f(x)dx = 8$ and $\displaystyle\int_3^7 f(x)dx = -4$

a) $\displaystyle\int_0^3 f(x)dx = 4$ b) $\displaystyle\int_0^3 f(x)dx = 12$

c) $\displaystyle\int_3^0 f(x)dx = 12$ d) $\displaystyle\int_7^3 f(x)dx = -1$

20. _____ Determine which of the following is **not** equal to $\displaystyle\int_1^8 axf(x)dx$

a) $\displaystyle -\int_8^1 axf(x)dx$ b) $\displaystyle a\int_1^8 xf(x)dx$

c) $\displaystyle x\int_1^8 af(x)dx$ d) $\displaystyle \int_1^5 axf(x)dx + \int_5^8 axf(x)dx$

21. _____ Find the average value of $f(x) = 4x + \cos x$ on the interval $[0, \pi]$

22. Evaluate $\dfrac{d}{dx}\displaystyle\int_{\pi/3}^{2x} 4\sin t\, dt$

23. Suppose a pebble has been dropped into a pond forming a circular ripple. If the rate of change of the circumference of the circle is 4π in/s, what is the rate of change of the area of the circle?

24. Determine if the function
$$f(x) = \begin{cases} 2x - 6 & \text{if } x \le 4 \\ x^2 - 5x + 6 & \text{if } x > 4 \end{cases}$$
is continuous. If the function is not continuous, explain why not.

25. Integrate the following: $\displaystyle\int \frac{3x - 6}{\sqrt{x^2 - 4x}}\,dx$

26. If $f(x) = x^2 \cos^2 x$, then $f'(\pi) =$

27. Find the most simplified value of $f'(x)$ if
$$f(x) = \frac{3x^2 - 12x}{x - 4}$$

28. Integrate the following: $\displaystyle\int \frac{5x^4 - 3x^2 + 7}{2x^2}\,dx$

29. Suppose $h(x), f(x)$, and $g(x)$ are continuous functions on [a, b] such that $\lim_{x \to c} f(x) = 3$, $\lim_{x \to c} g(x) = 3$, and $f(x) \le h(x) \le g(x)$ for all x on [a, b]. What can we conclude about the $\lim_{x \to c} h(x)$?

Justify your answer

a) We cannot conclude anything, since the function value at $x = c$ doesn't affect the limit value.

b) $\lim_{x \to c} h(x) = 3$

c) There is not enough information to conclude anything.

d) $\lim_{x \to c} h(x) = c$

30. This graph shows the velocity of an object moving along the x-axis for $0 \le t \le 9$. What is the total distance traveled by the object for $0 \le t \le 9$?
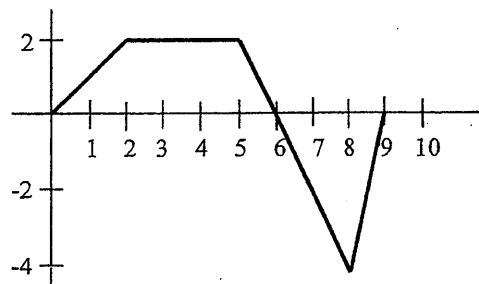
31. The width of a rectangle is decreasing at 4 cm/s and the height is increasing at 2 cm/s. Which of the following is true?
Justify your answer.
a) the area is always increasing
b) the area is always decreasing
c) the area is increasing when the width is greater than twice the height
d) the area is increasing when the height is greater than twice the width
e) the area is constant

32. A particle b is moving along the x-axis. Its velocity is given by $v(t) = t^3 - nt$, where $n > 0$. Which of the following are necessarily true?

I. b has a velocity of 0 when $t = \sqrt{n}$
II. b moves to the right for all t
III. b has a minimum speed when $t = \sqrt{n}$

a) I only   b) II only
c) III only   d) I and III
Justify your answer.

33. Evaluate $\displaystyle\lim_{x\to 0}\frac{4-4\cos 4x}{8x}$

34. Determine the intervals on which the function $f(x)=x^4-8x^3-72x^2+24x$ is concave up

35. Find the derivative of $f(x)=(2x^2-1)(\cos^2 x)$

36. Evaluate: $\displaystyle\int_{\pi/6}^{7\pi/6}\sec x\tan x\,dx$

37. Find the volume of the solid formed by revolving the region bounded by $f(x)=x^2$, $x=2$, and the x-axis around the line $y=4$.

38. A farmer has decided to build a pen for his horse alongside a drainage canal. If the pen is to be rectangular in shape and he has 400 feet of fencing available, what is the area of the largest possible pen he can create? He doesn't need to have fencing on the canal side of the pen.

39. What is the slope of the normal line to the curve $y = 4x^3 - 3x + 2$ at the point $(1, 3)$?	40. Find the area enclosed between $f(x) = x^2 - 4x + 3$ and the x-axis.
41. Evaluate $\lim\limits_{x \to \infty} \dfrac{5 - 4x^2}{9x - 3}$	42. If $y = 4\log_3 x$, find $\dfrac{dy}{dx}$
43. What is the area between $f(x) = (x-1)^2$ and $g(x) = 2x$ for $[1, 2]$?	44. Find the equation of the tangent line to the graph of $f(x) = 3x^2 - 6x + 12$ at the point where $x = 0$.

45. Find the absolute maximum and minimum values of $f(x)$ on the interval $[0, 4)$ if $f(x) = x^2 - 3x - 4$	46. Find the area under the curve $f(x) = x^2\sqrt{3x^3 + 3}$ between the x-intercepts of the function.
47. The velocity function of an object is $v(t) = 3t^2 - 2t - 1$, where v is in m/s. What is the total distance traveled in the interval $0 \le t \le 5$?	48. Evaluate $\int \dfrac{\ln(2x)}{2x}\,dx$
49. Find a general solution for the following differential equation: $y(x + 4) - y' = 0$	50. Find $\dfrac{d}{dx}(\arcsin(2x - 3))$

51. Evaluate $\int \dfrac{\sec^2(2x)}{\tan(2x)} dx$

52. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{\sqrt[3]{x}}{y}$

53. An object has a constant acceleration of 80 feet per second squared, an initial velocity of 18 feet per second, and an initial position of 10 feet. Find the position function describing the motion of this object.

54. Determine if the function

$$f(x) = \begin{cases} \dfrac{x^3 + 8}{x^2 - 4} & if\ x \neq -2 \\ 3 & if\ x = -2 \end{cases}$$ is continuous. If the function is not continuous, explain why not.

55. Evaluate $\displaystyle\lim_{x \to \infty} \dfrac{4x}{x^2 - 3}$

56. A radioactive element has a half-life of 32 days. What percentage of the original sample will be left after 45 days?

57. The graph shows the velocity of an object moving along the x-axis for $0 \leq t \leq 9$. During which of the following time intervals does the object have the greatest acceleration? Justify your answer
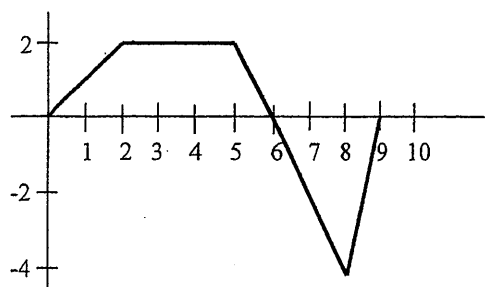
a) $2 < t < 5$ b) $5 < t < 8$ c) $t = 8$
d) $8 < t < 9$ e) none of these

---

58. The acceleration of an object moving along the x-axis is given by $a(t) = 18t - 2$, where the velocity is 12 when $t = 2$ and the position is 2 when $t = 1$. The position $x(t) =$

---

59. The rate of change in sales S is inversely proportional to the time, t, $t > 1$ measured in weeks. Find S as a function of t if sales after two weeks and six weeks are 250 units and 375 units respectively.

---

60. A 15-foot ladder is leaning against a house. If the tip of the ladder begins to slide down the house at a rate of 5 feet per second, what is the rate of change of the base of the ladder when the base is 9 feet from the house?

If R is the region in the first quadrant bounded by the y-axis and the graphs of the functions
$y = 5 - x^2$ and $y = 2 + 2\sin x$

a) Find the area of R

b) Find the volume of the solid formed when the area R is rotated about the x-axis.

c) Find the volume of the solid formed with base R, which has square cross sections perpendicular to the x-axis.

A square is inscribed in a circle so that the vertices of the square lie on the circumference of the circle. As the square expands, the circle expands so that the vertices of the square still lie on the circle. The perimeter of the square is increasing at a rate of 10 cm per second.

a) Find the rate at which the circumference of the circle is increasing.

b) At the instant when the area of the square is $25\ cm^2$, find the rate at which the area between the circle and the square is increasing.

c) If the square was replaced with an equilateral triangle whose perimeter was also increasing at a rate of 10cm per second, how would your answers to parts A0 and B0 be affected?

$y(t)$ is the temperature of a cup of tea, in degrees Fahrenheit, at time t minutes, $t \geq 0$.

Room temperature is $65°$ and the original temperature of the tea is $190°$.

The tea's temperature at time t is given by equation $\dfrac{dy}{dt} = -0.1(y - 65)$, with the initial condition $y(0) = 190$.

a) Find an expression for the temperature $y(t)$ in terms of time.

b) What is the temperature of the cup of tea after 12 minutes?

c) Tea is deemed safe to drink when its temperature is less than $130°$, at what time will the tea be cool enough to drink?

Bob is riding his bike along a straight road from P to Q, leaving point P at time t = 0 minutes. THE graph of Bob's velocity is shown below.

v(km/min)
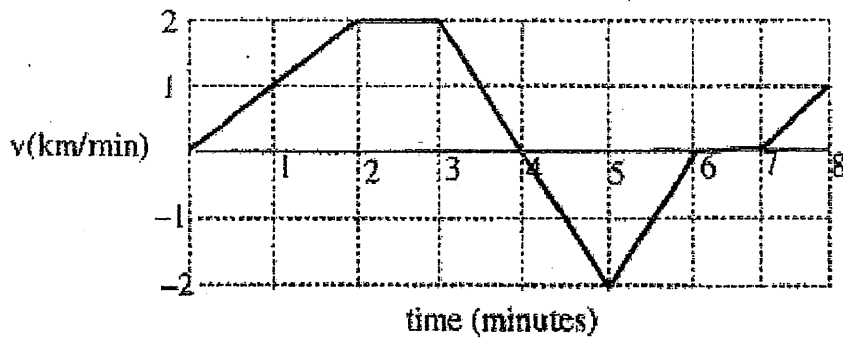
time (minutes)

a) How far away from point P is Bob after 6 minutes?
b) When does Bob change direction? Justify your answer.
c) Sketch a labelled graph of Bob's position against time.
d) Sketch a labelled graph of Bob's acceleration against time.

FRQ#5   No Calculators

The graph of the function f is shown to the right.

a. Find $\lim\limits_{h \to 0^-} \dfrac{f(0+h)-f(0)}{h}$

b. Find $\lim\limits_{x \to 0} f(x)$

c. Find $\lim\limits_{x \to 0} f'(x)$

d. Find $\displaystyle\int_{-1}^{0} f(x)\,dx$

e. Using a trapezoidal Riemann Sum with 3 sub-intervals approximate $\displaystyle\int_{-2}^{4} f(x)\,dx$

graph of the function f
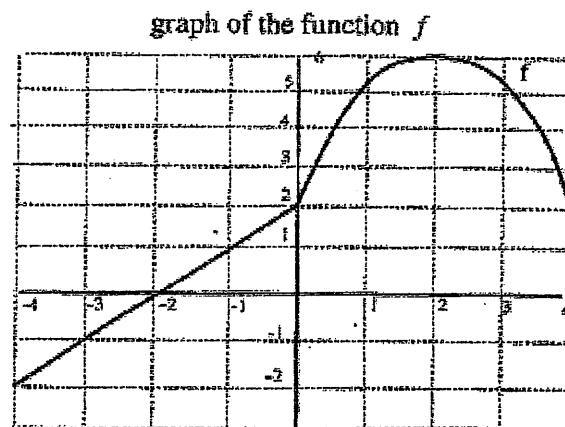
Given $y = \ln\left(\dfrac{x}{x+1}\right)$

a) What is the domain of y?

b) Find $\dfrac{dy}{dx}$

c) Find $\dfrac{d^2y}{dx^2}$

d) Find the equation of the line tangent to the graph of y at the point when x = 1.

e) Write an expression for the <u>derivative of the inverse</u> function for y.

_____   _____
Signature   Date