Name: ________________________________ Date: ___________ Period:_____ (1)

BC Calculus Unit 9 Parametric and Polar Test Review WS #1

Calculators Allowed: Show all work that lead to your answer to earn full credit.

1. A curve is defined parametrically by $x(t) = t^3 - 3t^2 + 4$ and $y(t) = \sqrt{t^2 + 16}$. What is the equation of the tangent line at the point defined by $t = 3$?

2. An object moves in the xy-plane so that its position at any time t is given by the parametric equations $x(t) = t^2 + 3$ and $y(t) = t^3 + 5t$. What is the rate of change of y with respect to x when $t = 1$?

3. A curve in the xy-plane is defined by $(x(t), y(t))$, where $x(t) = 3t$ and $y(t) = t^2 + 1$ for $t \geq 0$. What is $\frac{d^2y}{dx^2}$ in terms of t?

4. If $x(\theta) = \cot\theta$ and $y(\theta) = \csc\theta$, what is $\frac{d^2y}{dx^2}$ in terms of θ?

5. What is the length of the curve defined by the parametric equations $x(t) = 7 + 4t$ and $y(t) = 6 - t$ for the interval $0 \leq t \leq 9$?

(2)

6. What is the length of the curve defined by the parametric equations $x(\theta) = 3\cos 2\theta$ and $y(\theta) = 3\sin 2\theta$ for the interval $0 \le \theta \le \frac{\pi}{2}$?

7. If f is a vector-valued function defined by $\langle 2t^3 + 3t^2 + 4t + 1,\ t^3 - 4t - 1\rangle$ then $f''(2) =$

8. At time t, $0 \le t \le 2\pi$, the position of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle e^t \sin 3t,\ e^t \cos 3t\rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{6}$.

9. Find the vector-valued function $f(t)$ that satisfies the initial conditions $f(0) = \langle -2, 5\rangle$ and $f'(t) = \langle 10t^4,\ 2t\rangle$.

10. **Calculator active**: For $t \ge 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$ the particle is at position $(3, 4)$. It is known that $\frac{dx}{dt} = \sin 2t$ and $\frac{dy}{dt} = \frac{\sqrt{t}}{e^{2t}}$. Find the y-coordinate of the particles position at time $t = 3$.

11. A particle moving in the xy-plane has position given by parametric equations $x(t) = t$ and $y(t) = 4 - t^2$.
    A. Find the velocity vector.

    B. Find the speed when $t = 1$.

    C. Find the acceleration vector.

12. It is known the acceleration vector for a particle moving in the xy-plane is given by $a(t) = \langle t, \sin t\rangle$. When $t = 0$, the velocity vector $v(0) = \langle 0, -1\rangle$ and the position vector $p(0) = \langle 0, 0\rangle$. Find the position vector at time $t = 2$.

13. Find the slope of the tangent line to the polar curve $r = 2\cos 4\theta$ at the point where $\theta = \frac{\pi}{4}$.

14. **Calculator active.** For a certain polar curve $r = f(\theta)$, it is known that $\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta$ and $\frac{dy}{d\theta} = \sin\theta + \theta\cos\theta$. What is the value of $\frac{d^2y}{dx^2}$ at $\theta = 6$?

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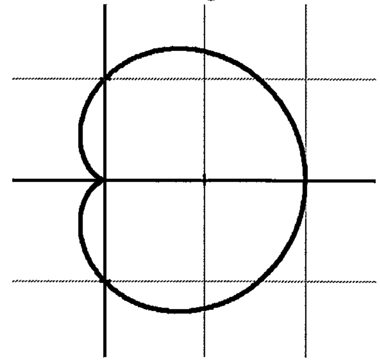
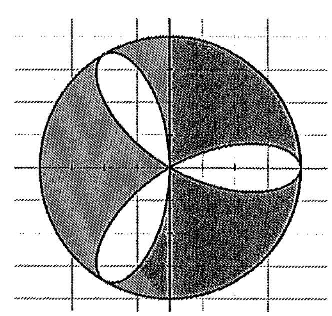
15. **Calculator active.** Find the total area enclosed by the polar curve $r = 1 + \cos\theta$ shown in the figure above.



16. **Calculator active.** Find the area of the inner loop of the polar curve $r = 3 - 6\sin\theta$.

17. Find the total area of the common interior of the polar graphs $r = 5 - 3\sin\theta$ and $r = 5 - 3\cos\theta$.

18. **Calculator active.** The figure shows the graphs of the polar curves $r = 4\cos 3\theta$ and $r = 4$. What is the sum of the areas of the shaded regions?



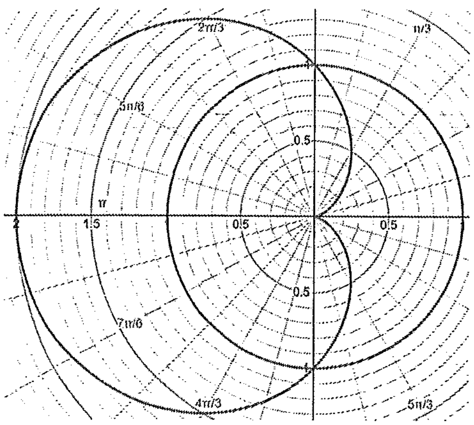
Name: ______________________________ Date: ____________ Period: _____

BC Calculus Unit 9 Parametric & Polar Test Review WS #2

Calculators Allowed: Show all work that lead to your answer to earn full credit.

1. What is the length of the curved defined by the parametric equations $x(t) = 9\cos t$ and $y(t) = 9\sin t$ for the interval $0 \le t \le 2\pi$?

2. **Calculator active.** Find the area of the region inside the circle $r = 1$ and outside the cardiod $r = 1 - \cos\theta$.



3. If $x(t) = 2t^3$ and $y(t) = t^3 - t$, what is $\frac{d^2y}{dx^2}$ in terms of t?

4. The position of a remote-controlled vehicle moving along a flat surface at time t is given by $(x(t), y(t))$, with velocity vector $v(t) = \langle 3t^2, 2t\rangle$ for $0 \le t \le 3$. Both $x(t)$ and $y(t)$ are measured in meters, and time t is in seconds. When $t = 0$, the remote-controlled vehicle is at the point $(1, 2)$.

   a. Find the acceleration vector of the remote-controlled vehicle when $t = 2$.

   b. Find the position of the remote-controlled vehicle when $t = 3$.

⑥

5. Which of the following gives the length of the path described by the parametric equations $x = 2e^{3t}$ and $y = 3t^2 + t$ from $0 \leq t \leq 1$?

A. $\int_0^1 \sqrt{12e^{6t} + (6t+1)^2}\, dt$

B. $\int_0^1 \sqrt{4e^{6t} + (6t+1)^2}\, dt$

C. $\int_0^1 \sqrt{4e^{6t} + 9t^4 + t^2}\, dt$

D. $\int_0^1 \sqrt{36e^{6t} + (6t+1)^2}\, dt$

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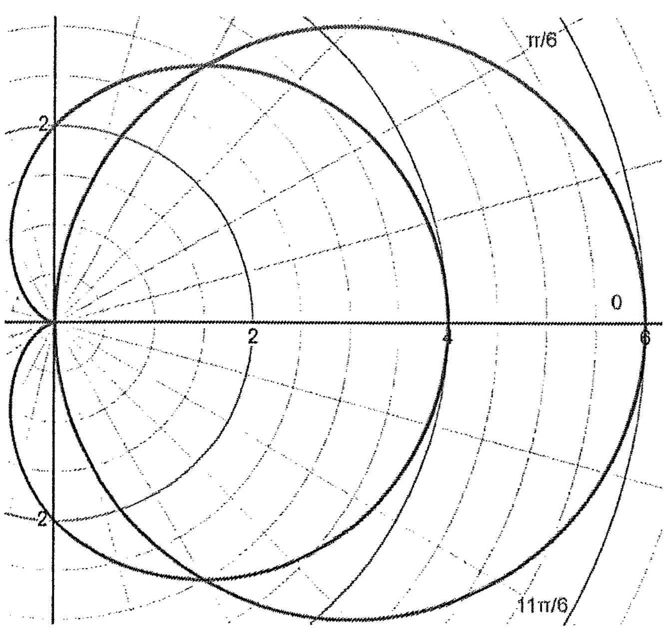
6. **Calculator active.** A polar curve is given by $r = \frac{5}{3-\sin\theta}$. What angle θ corresponds on the curve with a y-coordinate of -1?

7. If f is a vector-valued function defined by $\langle te^t,\ 2t^2e^t \rangle$ then $f''(1) = ?$

8. **Calculator active.** Find the area of the region common to the two regions bounded by the curves $r = 6\cos\theta$ and $r = 2 + 2\cos\theta$.



9. Find the vector-valued function $f(t)$ that satisfies the initial conditions $f(0) = \langle 3, 0\rangle$, and $f'(t) = \langle 4\sin\frac{t}{2}, -2\cos 2t\rangle$.

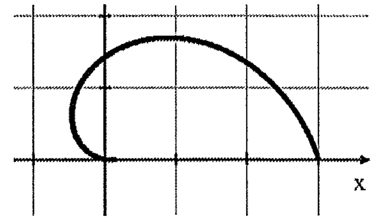
10. If $x = 7\cos\theta$ and $y = 7\sin\theta$, find the slope and the concavity at $\theta = \frac{\pi}{4}$.

11. **Calculator active.** At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle 9t^2, e^t\rangle$. If the particle is at point $(3, 4)$ at time $t = 0$, how far is the particle from the origin at time $t = 2$?

12. Find the slope of the tangent line to the polar curve $r = 2\cos\theta - 1$ at the point where $\theta = \frac{3\pi}{2}$.

13. Find the slope of the tangent line to the curve defined parametrically by $x(t) = 2\cos t$ and $y(t) = 3\sin^2 t$ at $t = \frac{\pi}{3}$.

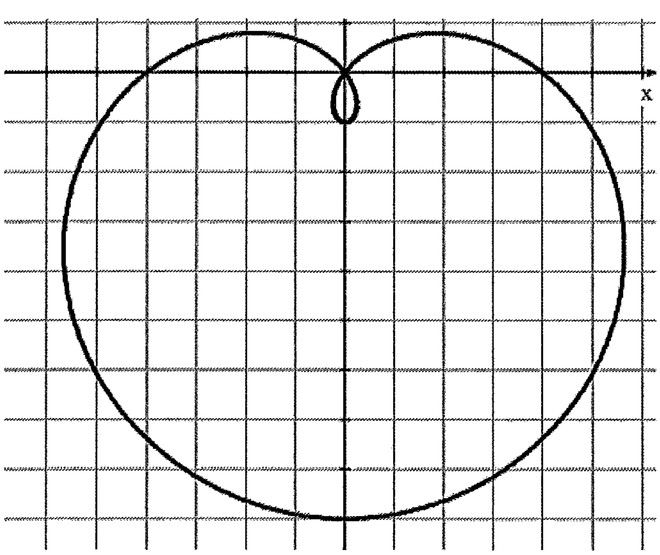
14. **Calculator active.** The graph shows the polar curve $r = 3 - \theta$ for $0 \leq \theta \leq \pi$. What is the area of the region bounded by the curve and the x-axis?



15. At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle \cos 2t, \sin 4t \rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{4}$.

16. Find an equation for the line tangent to the curve given by the parametric equations $x(t) = t^2 + 1$ and $y(t) = t^3 + t + 1$, when $t = 2$.

17. **Calculator active.** Find the total area enclose by the inner loop of the polar curve $r = 4 - 5\sin\theta$, shown in the figure.



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BC Calculus Unit 9 Parametric and Polar Test Review WS #3

Calculators Allowed: Show all work that lead to your answer to earn full credit.

1) What is the slope of the tangent line to the curve defined parametrically by $x(t) = \sqrt{t}$ and $y(t) = \frac{1}{4}(t^2 - 4)$, $t \geq 0$ at the point (2,3)?

2) If $x = \sin\theta$ and $y = 2\cos\theta$, what is $\frac{d^2y}{dx^2}$ in terms of θ?

3) Which of the following gives the length of the path described by the parametric equations $x = e^{2t}$ and $y = 1 - 2t$ from $0 \leq t \leq 3$?

A. $\int_0^3 \sqrt{4e^{2t} + 4}\,dt$ B. $\int_0^3 \sqrt{2e^{2t} + 2}\,dt$

C. $\int_0^3 \sqrt{4e^{4t} + 4}\,dt$ D. $\int_0^3 \sqrt{e^{4t} + 4}\,dt$

4) The position of a particle moving in the xy-plane is defined by the vector-valued function, $f(t) = \langle t^3 - 9t^2 + 1,\ 2t^3 - 15t^2 - 36t + 1\rangle$. For what value of t is the particle at rest?

5) At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle e^{2t}\cos t,$ $e^{2t}\sin t \rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

6) **Calculator active.** At time $t \geq 0$, a particle moving in the xy-plane has a velocity vector given by $v(t) = \langle 2, 2^{-t^2} \rangle$. If the particle is at point $\left(1, \frac{1}{2}\right)$ at time $t = 0$, how far is the particle from the origin at time $t = 1$?

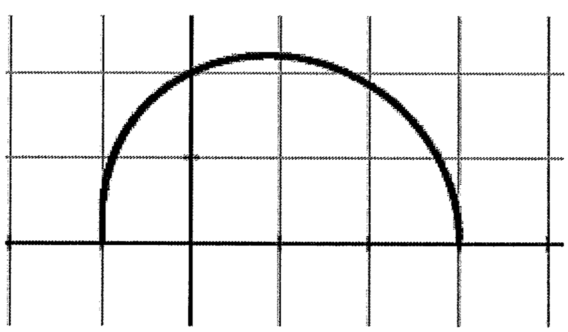
7) **Calculator active.** The position of a particle at time $t \geq 0$ is given by $x(t) = \frac{\sqrt{t+1}}{3}$ and $y(t) = t^2 + 1$. Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

8) **Calculator active.** The velocity vector a particle moving in the xy-plane has components given by $\frac{dx}{dt} = \sin 2t$ and $\frac{dy}{dt} = e^{\cos t}$. At time $t = 2$, the position of the particle is $(3, 2)$. What is the x-coordinate of the position vector at time $t = 3$?

9) A particle moves along the polar curve $r = 4 - 2\cos\theta$ so that $\frac{d\theta}{dt} = 4$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$.

10) **Calculator active.** For a certain polar curve $r = f(\theta)$, it is known that $\frac{dx}{d\theta} = 3\cos\theta - 3\theta\sin\theta$ and $\frac{dy}{d\theta} = 3(\sin\theta + \theta\cos\theta)$. What is the value of $\frac{d^2y}{dx^2}$ at $\theta = 3$?

11) The graph to the right shows the polar curve $r = 2 + \cos\theta$ for $0 \leq \theta \leq \pi$. What is the area of the region bounded by the curve and the x-axis?



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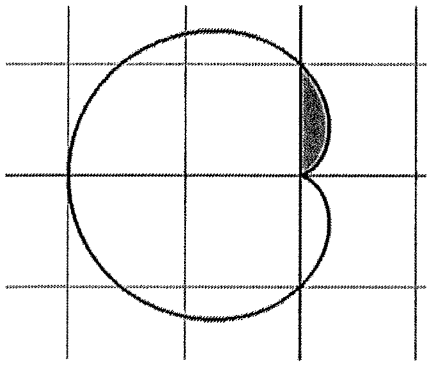
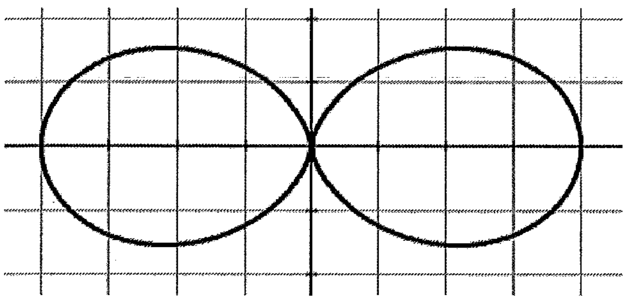
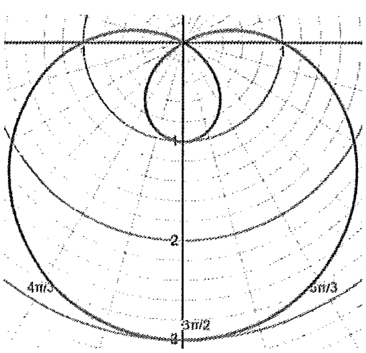
12) Find the area of the shaded region for the polar curve $r = 1 - \cos\theta$.



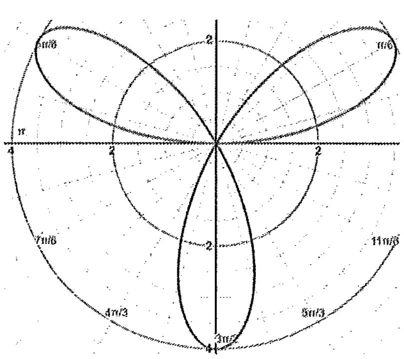
13) Find the total area enclosed by the polar curve $r = 2 + 2\cos 2\theta$ shown in the figure



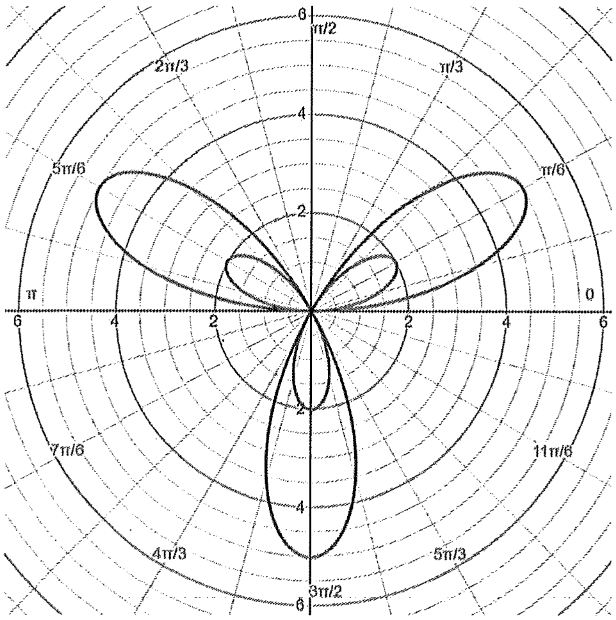
14) Write do not solve, an integral expression that represents the area enclosed by the smaller loop of the polar curve $r = 1 - 2\sin\theta$.



15) Find the limits of integration required to find the area of one petal of the polar graph $r = 4\sin 3\theta$ in the second quadrant.

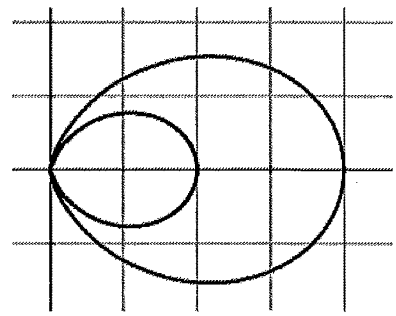


16) What is the total area between the polar curves $r = 2\sin 3\theta$ and $r = 5\sin 3\theta$.



17)

The figure to the right shows the graphs of the polar curves $r = 2\cos^2\theta$ and $r = 4\cos^2\theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Which of the following integrals gives the area of the region bounded between the two polar curves?



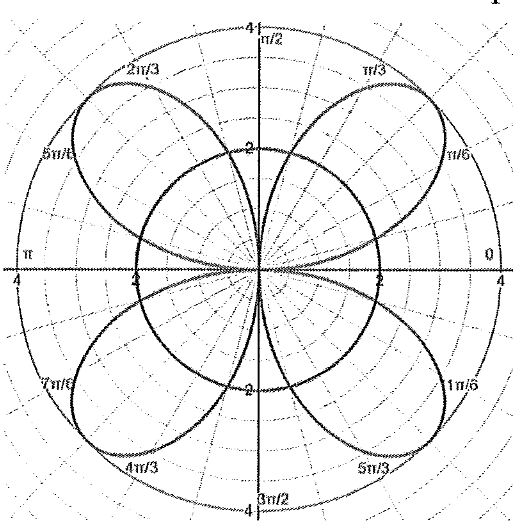
A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta$

B. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6\cos^4\theta \, d\theta$

C. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos^4\theta \, d\theta$

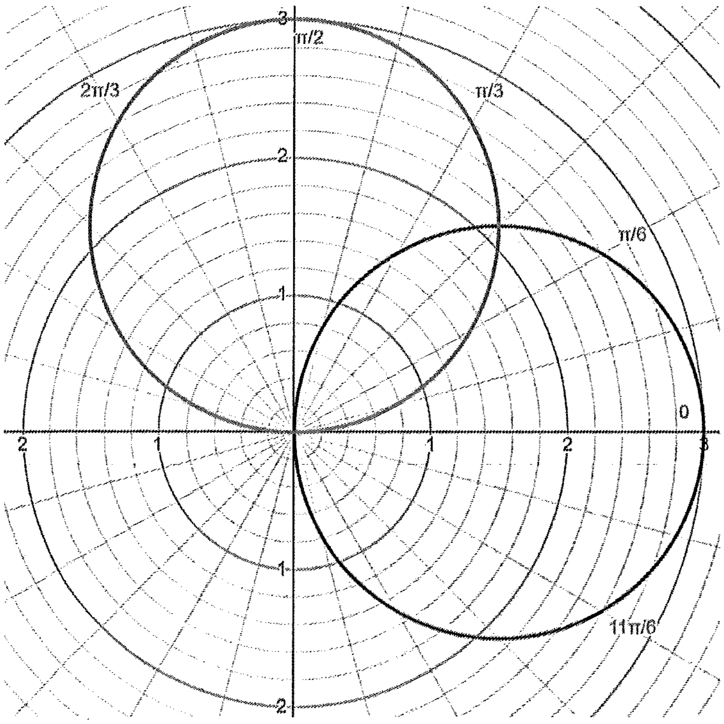
D. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos^2\theta \, d\theta$

18) Find the total area in the first quadrant of the common interior of $r = 4\sin 2\theta$ and $r = 2$.



(14)

19) Find the area of the common interior of the polar graphs $r = 3\cos\theta$ and $r = 3\sin\theta$.



20)

Let S be the region in the 1st Quadrant bounded above by the graph of the polar curve $r = \cos\theta$ and bounded below by the graph of the polar curve $r = \frac{7}{2}\theta$, as shown in the figure. The two curves intersect when $\theta = 0.275$. What is the area of S?