

Name: _____ Date: _____ Period: _____

①

BC Calculus Unit 9 Parametric and Polar Test Review WS #1

Calculators Allowed: Show all work that lead to your answer to earn full credit.

1. A curve is defined parametrically by $x(t) = t^3 - 3t^2 + 4$ and $y(t) = \sqrt{t^2 + 16}$. What is the equation of the tangent line at the point defined by $t = 3$?
2. An object moves in the xy -plane so that its position at any time t is given by the parametric equations $x(t) = t^2 + 3$ and $y(t) = t^3 + 5t$. What is the rate of change of y with respect to x when $t = 1$?
3. A curve in the xy -plane is defined by $(x(t), y(t))$, where $x(t) = 3t$ and $y(t) = t^2 + 1$ for $t \geq 0$. What is $\frac{d^2y}{dx^2}$ in terms of t ?
4. If $x(\theta) = \cot \theta$ and $y(\theta) = \csc \theta$, what is $\frac{d^2y}{dx^2}$ in terms of θ ?
5. What is the length of the curve defined by the parametric equations $x(t) = 7 + 4t$ and $y(t) = 6 - t$ for the interval $0 \leq t \leq 9$?

2

6. What is the length of the curve defined by the parametric equations $x(\theta) = 3 \cos 2\theta$ and $y(\theta) = 3 \sin 2\theta$ for the interval $0 \leq \theta \leq \frac{\pi}{2}$?
7. If f is a vector-valued function defined by $\langle 2t^3 + 3t^2 + 4t + 1, t^3 - 4t - 1 \rangle$ then $f''(2) =$
8. At time t , $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy -plane is given by the vector-valued function, $f(t) = \langle e^t \sin 3t, e^t \cos 3t \rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{6}$.
9. Find the vector-valued function $f(t)$ that satisfies the initial conditions $f(0) = \langle -2, 5 \rangle$ and $f'(t) = \langle 10t^4, 2t \rangle$.
10. **Calculator active:** For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$ the particle is at position $(3, 4)$. It is known that $\frac{dx}{dt} = \sin 2t$ and $\frac{dy}{dt} = \frac{\sqrt{t}}{e^{2t}}$. Find the y -coordinate of the particles position at time $t = 3$.

11. A particle moving in the xy -plane has position given by parametric equations $x(t) = t$ and $y(t) = 4 - t^2$.

A. Find the velocity vector.

B. Find the speed when $t = 1$.

C. Find the acceleration vector.

12. It is known the acceleration vector for a particle moving in the xy -plane is given by $a(t) = \langle t, \sin t \rangle$. When $t = 0$, the velocity vector $v(0) = \langle 0, -1 \rangle$ and the position vector $p(0) = \langle 0, 0 \rangle$. Find the position vector at time $t = 2$.

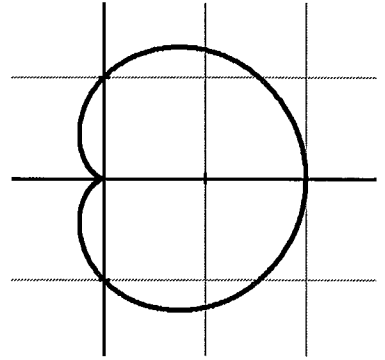
13. Find the slope of the tangent line to the polar curve $r = 2 \cos 4\theta$ at the point where $\theta = \frac{\pi}{4}$.

14. **Calculator active.** For a certain polar curve $r = f(\theta)$, it is known that $\frac{dx}{d\theta} = \cos \theta - \theta \sin \theta$ and

$\frac{dy}{d\theta} = \sin \theta + \theta \cos \theta$. What is the value of $\frac{d^2y}{dx^2}$ at $\theta = 6$?

4

15. **Calculator active.** Find the total area enclosed by the polar curve $r = 1 + \cos \theta$ shown in the figure above.



16. **Calculator active.** Find the area of the inner loop of the polar curve $r = 3 - 6 \sin \theta$.

17. Find the total area of the common interior of the polar graphs $r = 5 - 3 \sin \theta$ and $r = 5 - 3 \cos \theta$.

18. **Calculator active.** The figure shows the graphs of the polar curves $r = 4 \cos 3\theta$ and $r = 4$. What is the sum of the areas of the shaded regions?

