Name: _____ Date: _____ Period: _____

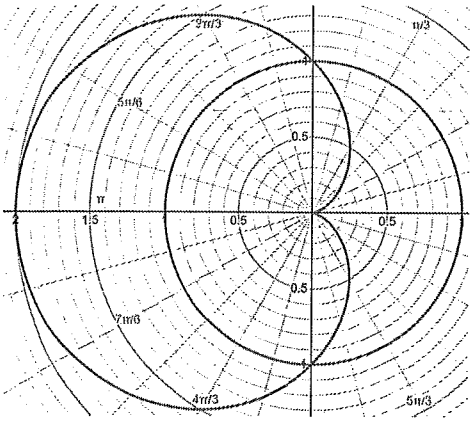
(5)

BC Calculus Unit 9 Parametric & Polar Test Review WS #2

Calculators Allowed: Show all work that lead to your answer to earn full credit.

1. What is the length of the curved defined by the parametric equations $x(t) = 9\cos t$ and $y(t) = 9\sin t$ for the interval $0 \leq t \leq 2\pi$?

2. **Calculator active.** Find the area of the region inside the circle $r = 1$ and outside the cardiod $r = 1 - \cos\theta$.



3. If $x(t) = 2t^3$ and $y(t) = t^3 - t$, what is $\frac{d^2y}{dx^2}$ in terms of t?

4. The position of a remote-controlled vehicle moving along a flat surface at time t is given by $(x(t), y(t))$, with velocity vector $v(t) = \langle 3t^2, 2t\rangle$ for $0 \leq t \leq 3$. Both $x(t)$ and $y(t)$ are measured in meters, and time t is in seconds. When $t = 0$, the remote-controlled vehicle is at the point $(1, 2)$.

   a. Find the acceleration vector of the remote-controlled vehicle when $t = 2$.

   b. Find the position of the remote-controlled vehicle when $t = 3$.

5. Which of the following gives the length of the path described by the parametric equations $x = 2e^{3t}$ and $y = 3t^2 + t$ from $0 \le t \le 1$?

A. $\int_0^1 \sqrt{12e^{6t} + (6t+1)^2}\, dt$

B. $\int_0^1 \sqrt{4e^{6t} + (6t+1)^2}\, dt$

C. $\int_0^1 \sqrt{4e^{6t} + 9t^4 + t^2}\, dt$

D. $\int_0^1 \sqrt{36e^{6t} + (6t+1)^2}\, dt$

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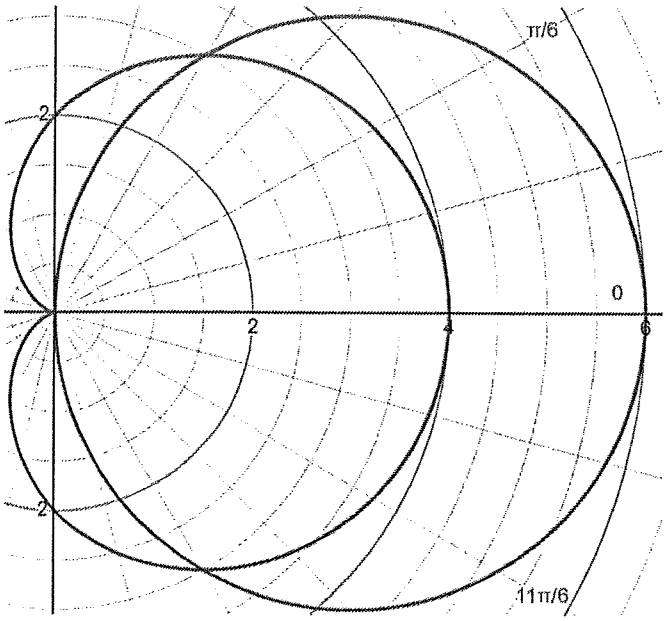
6. **Calculator active.** A polar curve is given by $r = \frac{5}{3-\sin\theta}$. What angle θ corresponds on the curve with a y-coordinate of -1?

7. If f is a vector-valued function defined by $\langle te^t,\ 2t^2e^t \rangle$ then $f''(1) =$?

8. **Calculator active.** Find the area of the region common to the two regions bounded by the curves $r = 6\cos\theta$ and $r = 2 + 2\cos\theta$.



9. Find the vector-valued function $f(t)$ that satisfies the initial conditions $f(0) = \langle 3, 0 \rangle$, and $f'(t) = \langle 4 \sin \frac{t}{2}, -2 \cos 2t \rangle$.

10. If $x = 7 \cos \theta$ and $y = 7 \sin \theta$, find the slope and the concavity at $\theta = \frac{\pi}{4}$.

11. **Calculator active.** At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle 9t^2, e^t \rangle$. If the particle is at point $(3, 4)$ at time $t = 0$, how far is the particle from the origin at time $t = 2$?

12. Find the slope of the tangent line to the polar curve $r = 2 \cos \theta - 1$ at the point where $\theta = \frac{3\pi}{2}$.

13. Find the slope of the tangent line to the curve defined parametrically by $x(t) = 2 \cos t$ and $y(t) = 3 \sin^2 t$ at $t = \frac{\pi}{3}$.

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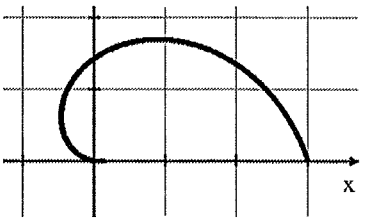
14. **Calculator active.** The graph shows the polar curve $r = 3 - \theta$ for $0 \leq \theta \leq \pi$. What is the area of the region bounded by the curve and the x-axis?



15. At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle \cos 2t, \sin 4t \rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{4}$.

16. Find an equation for the line tangent to the curve given by the parametric equations $x(t) = t^2 + 1$ and $y(t) = t^3 + t + 1$, when $t = 2$.

17. **Calculator active.** Find the total area enclose by the inner loop of the polar curve $r = 4 - 5 \sin \theta$, shown in the figure.