Name: _____ Date: _____ Period: _____ (9)

BC Calculus Unit 9 Parametric and Polar Test Review WS #3

Calculators Allowed: Show all work that lead to your answer to earn full credit.

1) What is the slope of the tangent line to the curve defined parametrically by $x(t) = \sqrt{t}$ and $y(t) = \frac{1}{4}(t^2 - 4)$, $t \geq 0$ at the point (2,3)?

2) If $x = \sin\theta$ and $y = 2\cos\theta$, what is $\frac{d^2y}{dx^2}$ in terms of θ?

3) Which of the following gives the length of the path described by the parametric equations $x = e^{2t}$ and $y = 1 - 2t$ from $0 \leq t \leq 3$?

A. $\int_0^3 \sqrt{4e^{2t} + 4}\, dt$ B. $\int_0^3 \sqrt{2e^{2t} + 2}\, dt$

C. $\int_0^3 \sqrt{4e^{4t} + 4}\, dt$ D. $\int_0^3 \sqrt{e^{4t} + 4}\, dt$

4) The position of a particle moving in the xy-plane is defined by the vector-valued function, $f(t) = \langle t^3 - 9t^2 + 1,\ 2t^3 - 15t^2 - 36t + 1 \rangle$. For what value of t is the particle at rest?

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5) At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle e^{2t} \cos t, e^{2t} \sin t \rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

6) **Calculator active.** At time $t \geq 0$, a particle moving in the xy-plane has a velocity vector given by $v(t) = \langle 2, 2^{-t^2} \rangle$. If the particle is at point $\left(1, \frac{1}{2}\right)$ at time $t = 0$, how far is the particle from the origin at time $t = 1$?

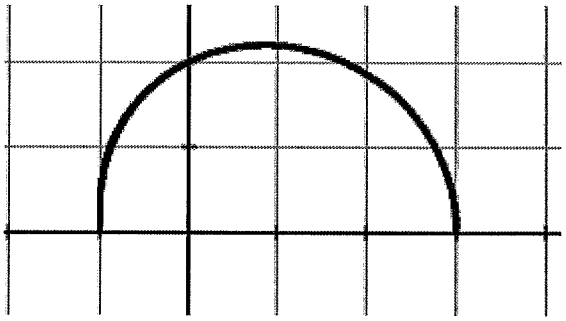
7) **Calculator active.** The position of a particle at time $t \geq 0$ is given by $x(t) = \frac{\sqrt{t+1}}{3}$ and $y(t) = t^2 + 1$. Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

8) **Calculator active.** The velocity vector a particle moving in the xy-plane has components given by $\frac{dx}{dt} = \sin 2t$ and $\frac{dy}{dt} = e^{\cos t}$. At time $t = 2$, the position of the particle is $(3, 2)$. What is the x-coordinate of the position vector at time $t = 3$?

9) A particle moves along the polar curve $r = 4 - 2\cos\theta$ so that $\frac{d\theta}{dt} = 4$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$.

10) **Calculator active.** For a certain polar curve $r = f(\theta)$, it is known that $\frac{dx}{d\theta} = 3\cos\theta - 3\theta\sin\theta$ and $\frac{dy}{d\theta} = 3(\sin\theta + \theta\cos\theta)$. What is the value of $\frac{d^2y}{dx^2}$ at $\theta = 3$?

11) The graph to the right shows the polar curve $r = 2 + \cos\theta$ for $0 \leq \theta \leq \pi$. What is the area of the region bounded by the curve and the x-axis?



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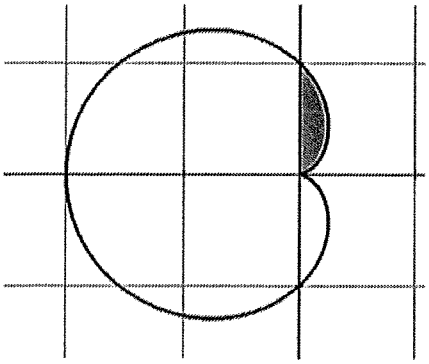
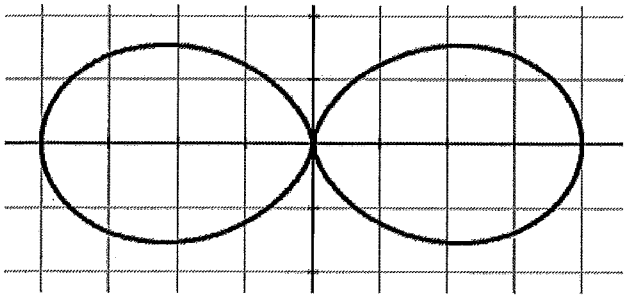
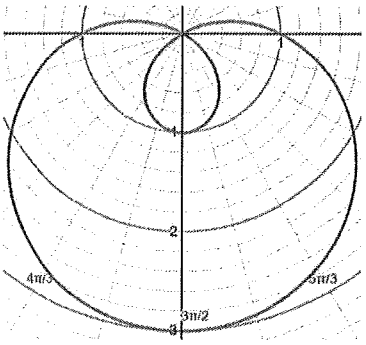
12) Find the area of the shaded region for the polar curve $r = 1 - \cos\theta$.



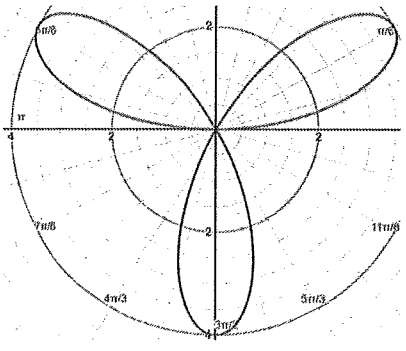
13) Find the total area enclosed by the polar curve $r = 2 + 2\cos 2\theta$ shown in the figure



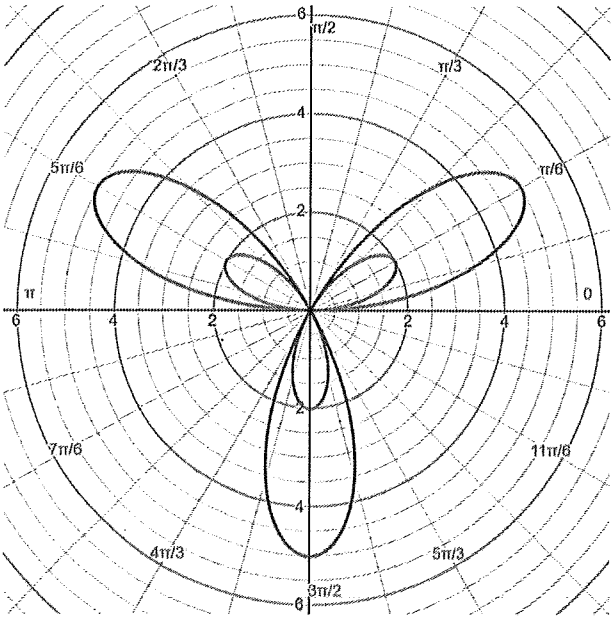
14) Write do not solve, an integral expression that represents the area enclosed by the smaller loop of the polar curve $r = 1 - 2\sin\theta$.



15) Find the limits of integration required to find the area of one petal of the polar graph $r = 4\sin 3\theta$ in the second quadrant.

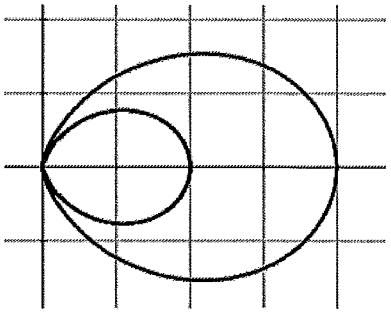


16) What is the total area between the polar curves $r = 2\sin 3\theta$ and $r = 5\sin 3\theta$.



17)

The figure to the right shows the graphs of the polar curves $r = 2\cos^2\theta$ and $r = 4\cos^2\theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Which of the following integrals gives the area of the region bounded between the two polar curves?



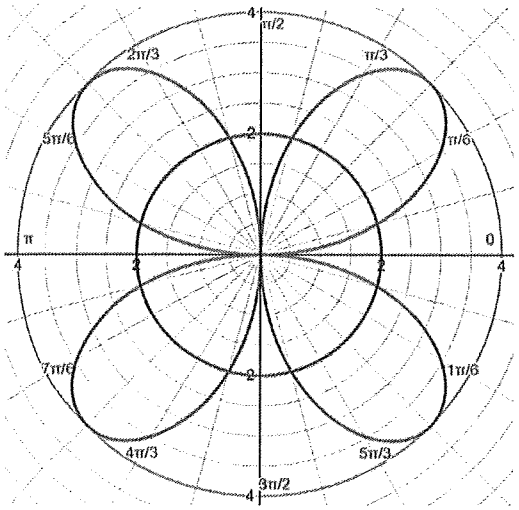
A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$

B. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6\cos^4\theta\, d\theta$

C. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos^4\theta\, d\theta$

D. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos^2\theta\, d\theta$

18) Find the total area in the first quadrant of the common interior of $r = 4\sin 2\theta$ and $r = 2$.



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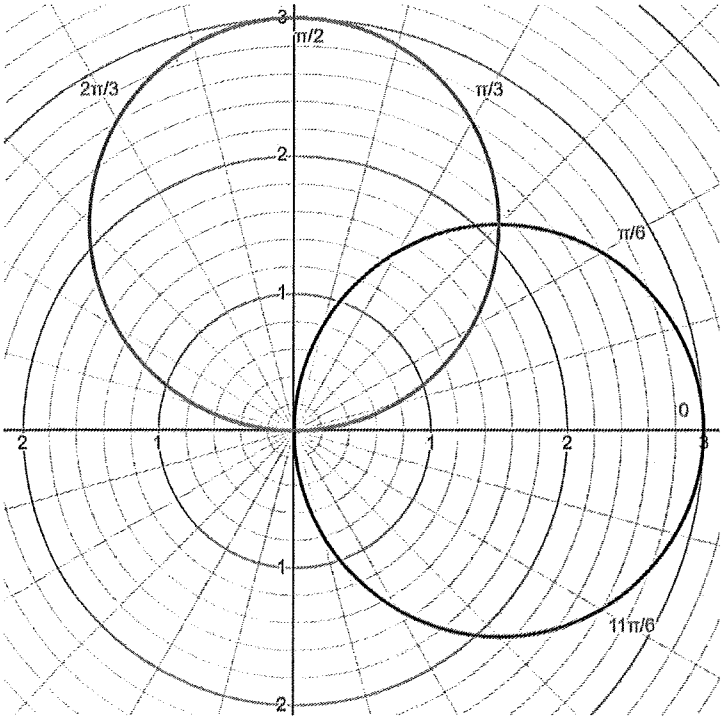
19) Find the area of the common interior of the polar graphs $r = 3\cos\theta$ and $r = 3\sin\theta$.



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Let S be the region in the 1st Quadrant bounded above by the graph of the polar curve $r = \cos\theta$ and bounded below by the graph of the polar curve $r = \frac{7}{2}\theta$, as shown in the figure. The two curves intersect when $\theta = 0.275$. What is the area of S?